Week 6: Wednesday

EECS 281

MAXIM ALEKSA

maximal.io

Split string

Interview Question

Given an input string and a dictionary of words, split the input string into a space-separated sequence of dictionary words if possible.

For example, if the input string is `"applepie"` and dictionary contains a standard set of English words, then we would return the string `"apple pie"` as output.

http://thenoisychannel.com/2011/08/08/retiring-a-great-interview-problem

Split string

Interview Question

Agenda

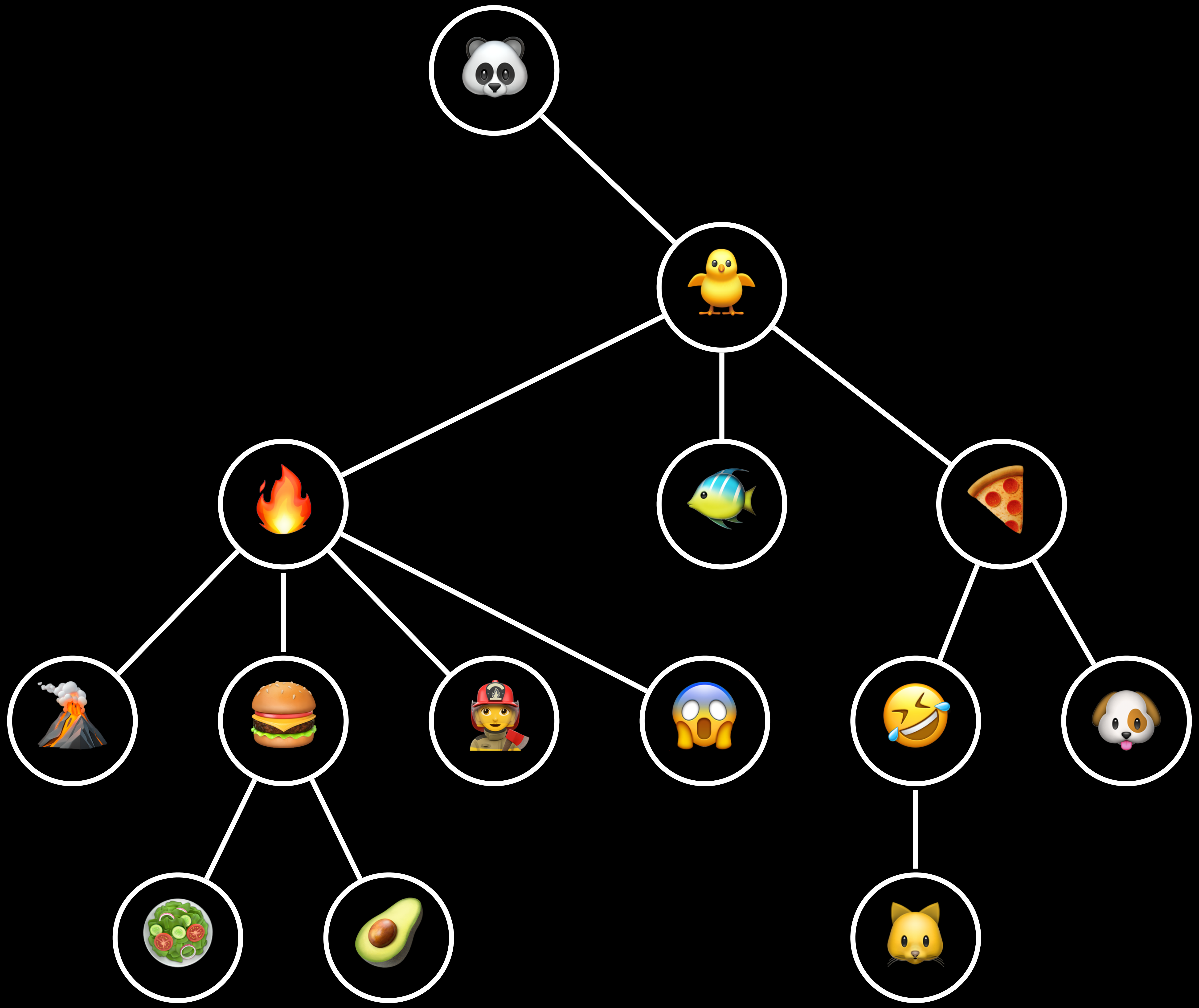
AVL Trees

Graphs

Graph Representations

Graph Traversals

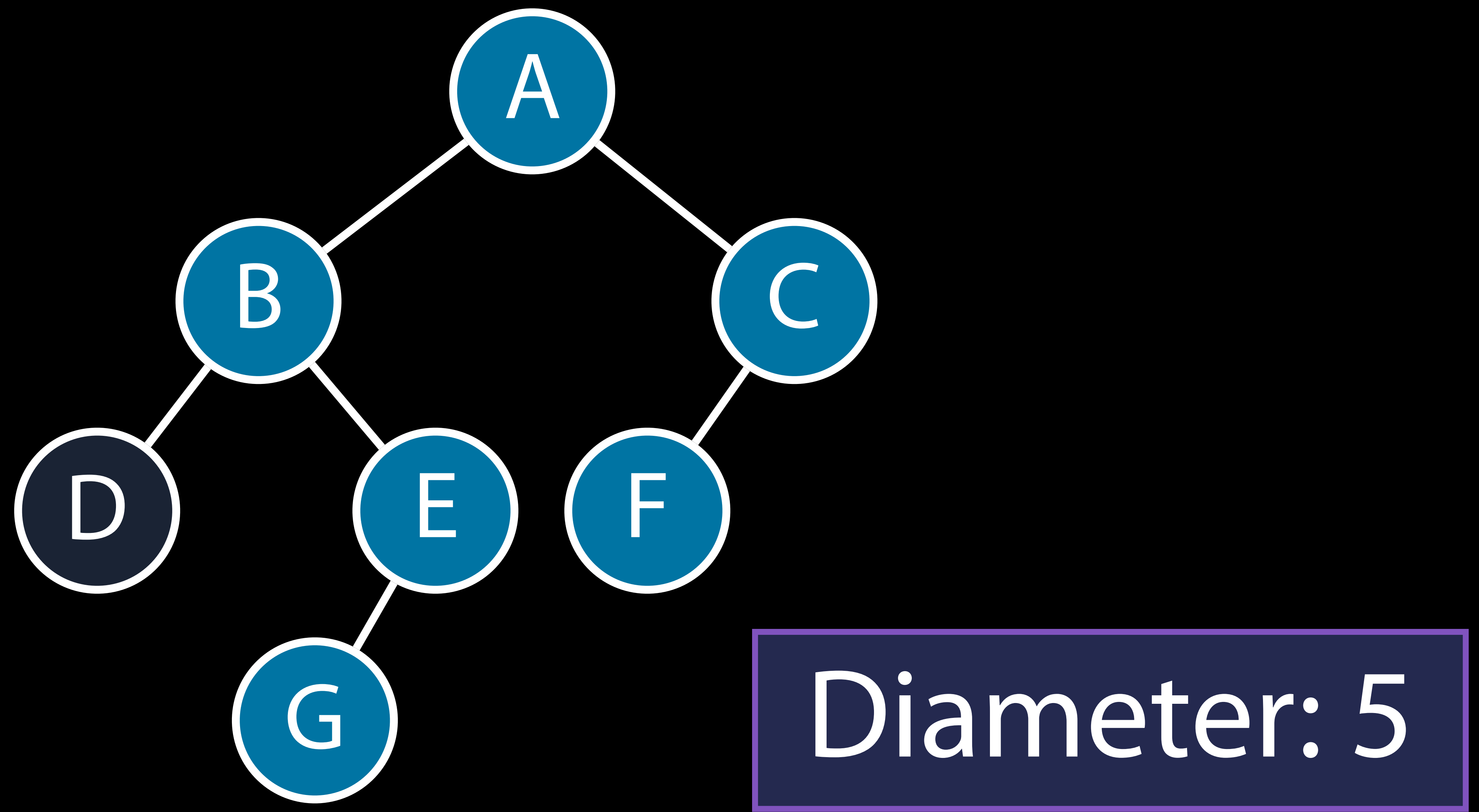
Trees

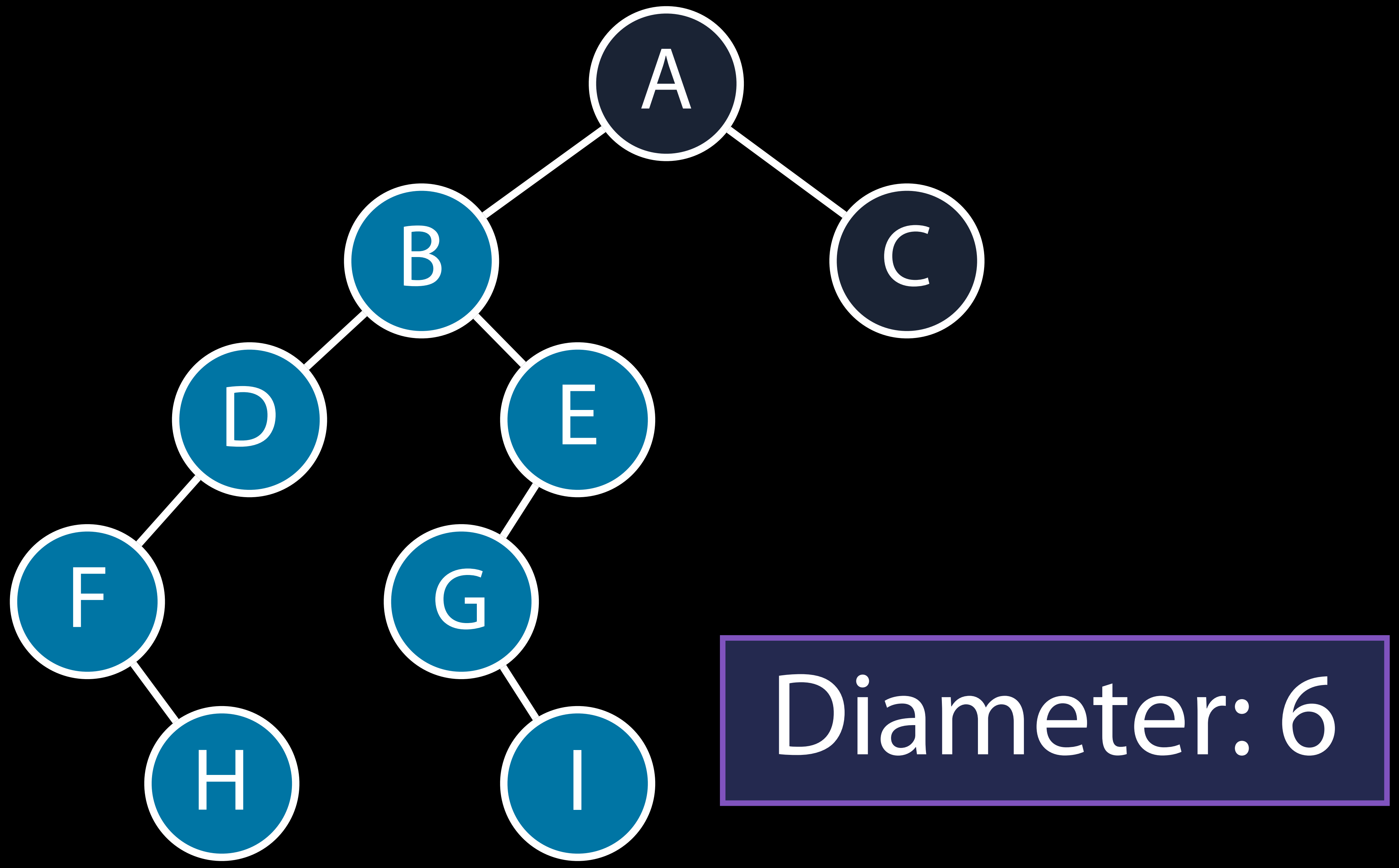


Tree diameter

The **diameter** of a tree is the maximum number of edges on any path connecting two nodes of the tree.

The diameter of an empty tree or a tree with one node is 0.





Tree Diameter

```
int height(const BinaryTreeNode* tree) {
    // represent absent nodes with height -1
    if (tree == nullptr) {
        return -1;
    } else {
        return max(height(tree->left), height(tree->right)) + 1;
    }
}

int diameter(const BinaryTreeNode* tree) {
    if (tree == nullptr) {
        return 0;
    } else {
        // check diameter of subtrees
        int childDiameter = max(diameter(tree->left), diameter(tree->right));
        int nodeDiameter = 2 + height(tree->left) + height(tree->right);
        return max(childDiameter, nodeDiameter);
    }
}
```

Tree diameter

```cpp
class BinaryTreeNode {
public:
    BinaryTreeNode* left = nullptr;
    BinaryTreeNode* right = nullptr;
    int value;
    BinaryTreeNode(int n) : value(n) {}
};
```

Balanced BSTs

Time complexity of find, insert, remove: $O(\log n)$

Balanced BSTs

AVL Trees

B-Trees/2-3-4 Trees

BB[α] Trees

Red-black Trees

Splay-Trees

Skip Lists

Scapegoat Trees

Treaps

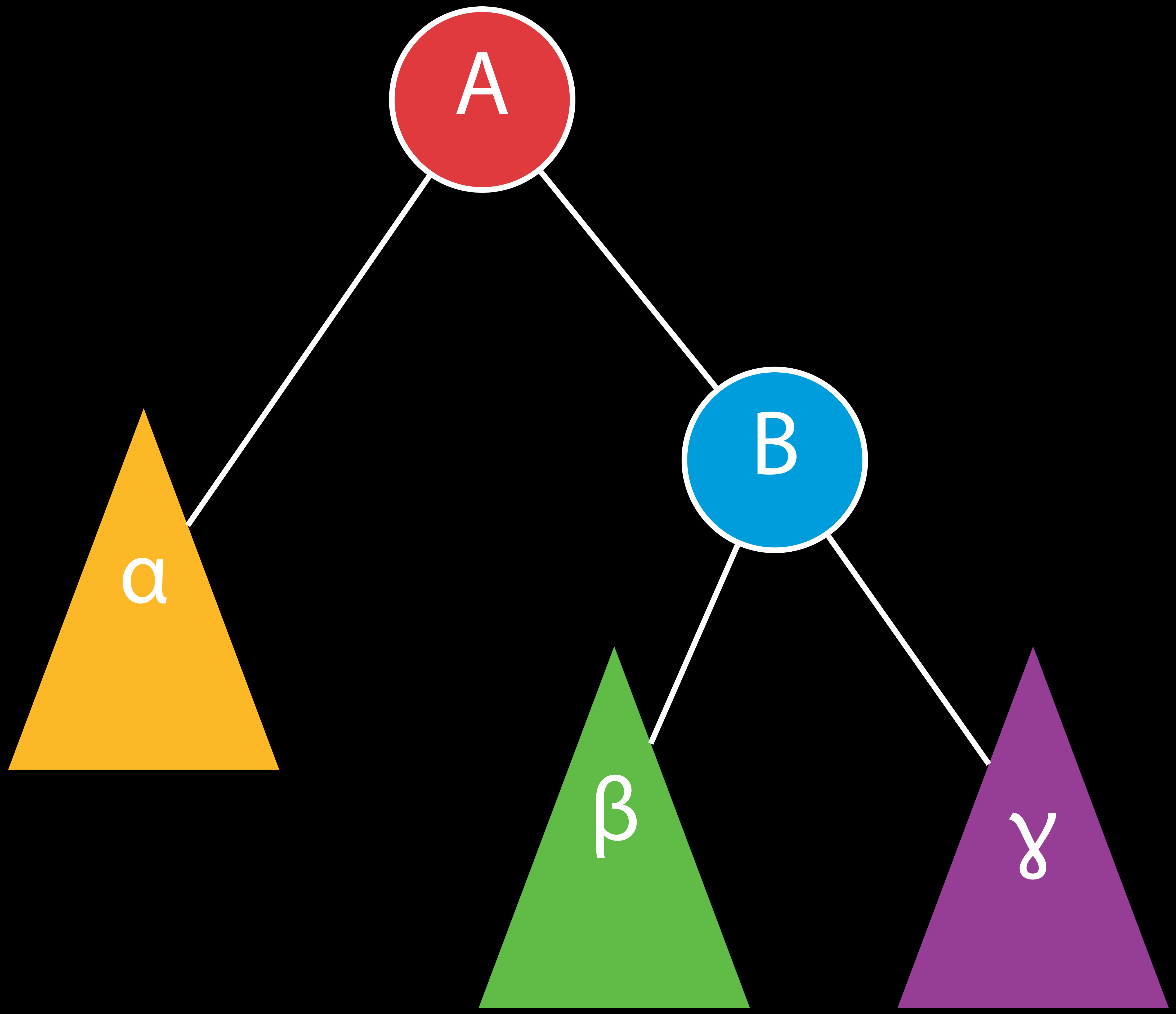
Definitions

Height of a node: max(height of left child, height of right child). Height of a leaf is 0.

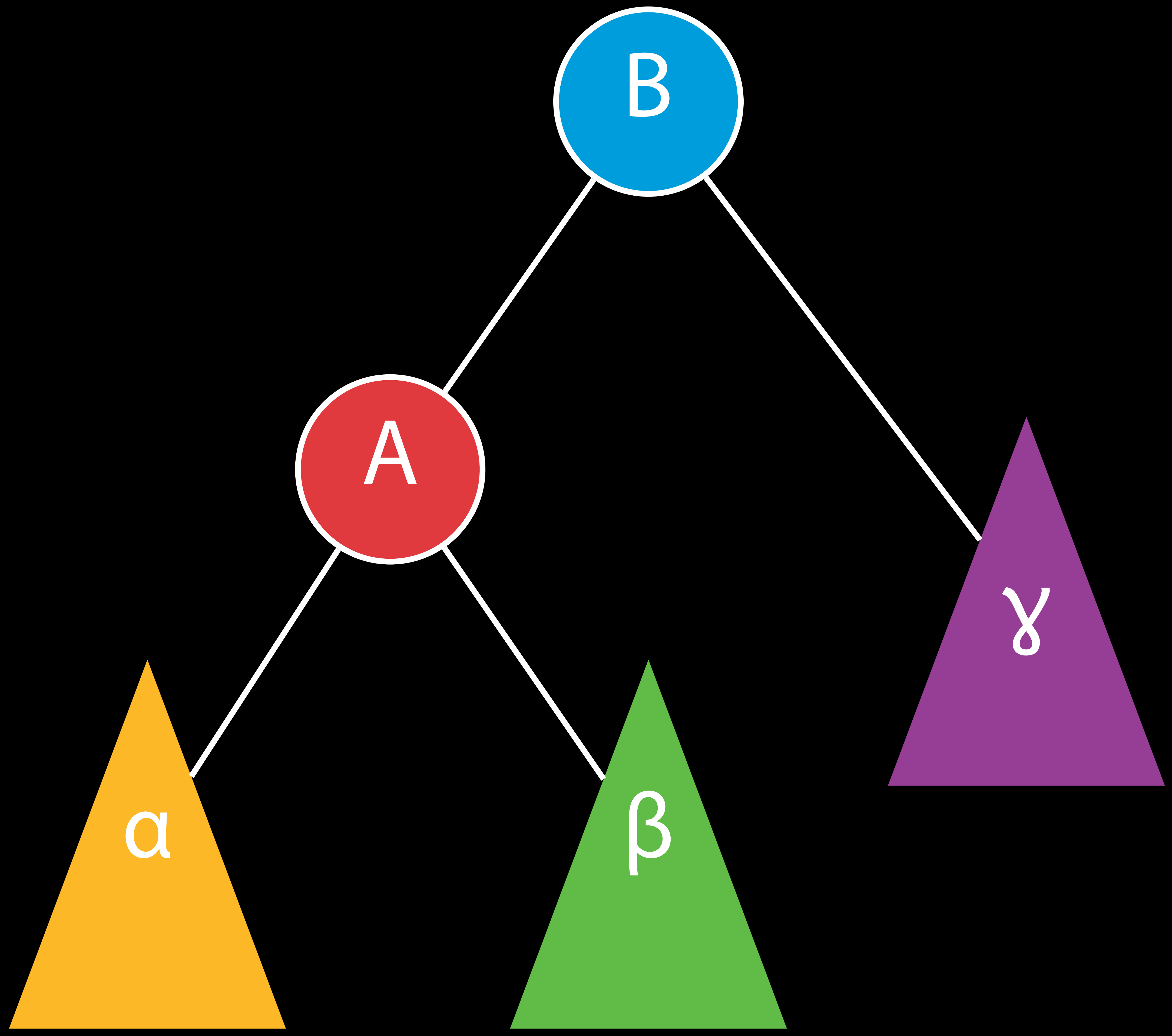
In-order predecessor: go to left subtree and find the right-most node.

In-order successor: go to right subtree and find the left-most node.

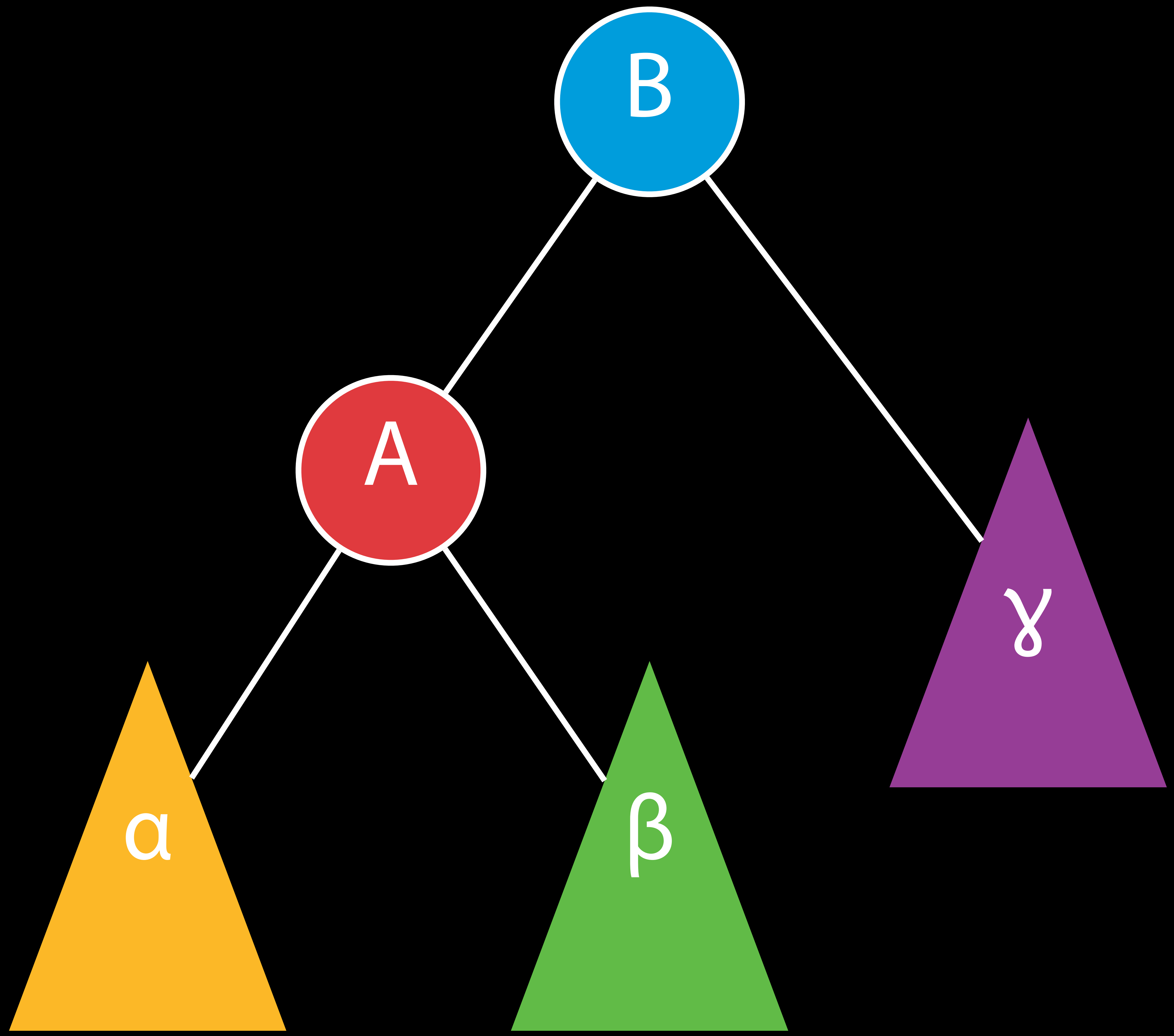
Left Rotation on A



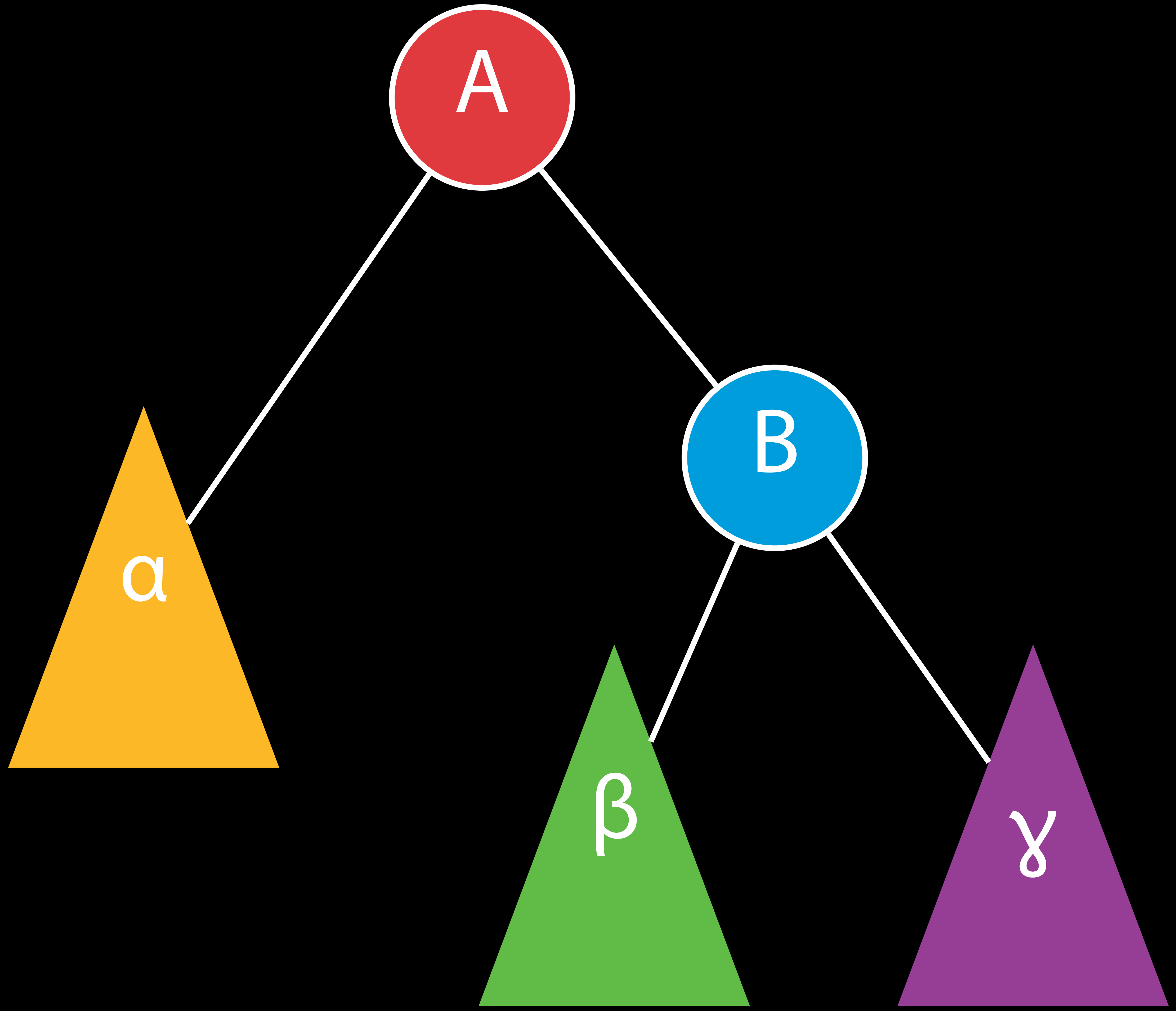
Left Rotation on A



Right Rotation on B

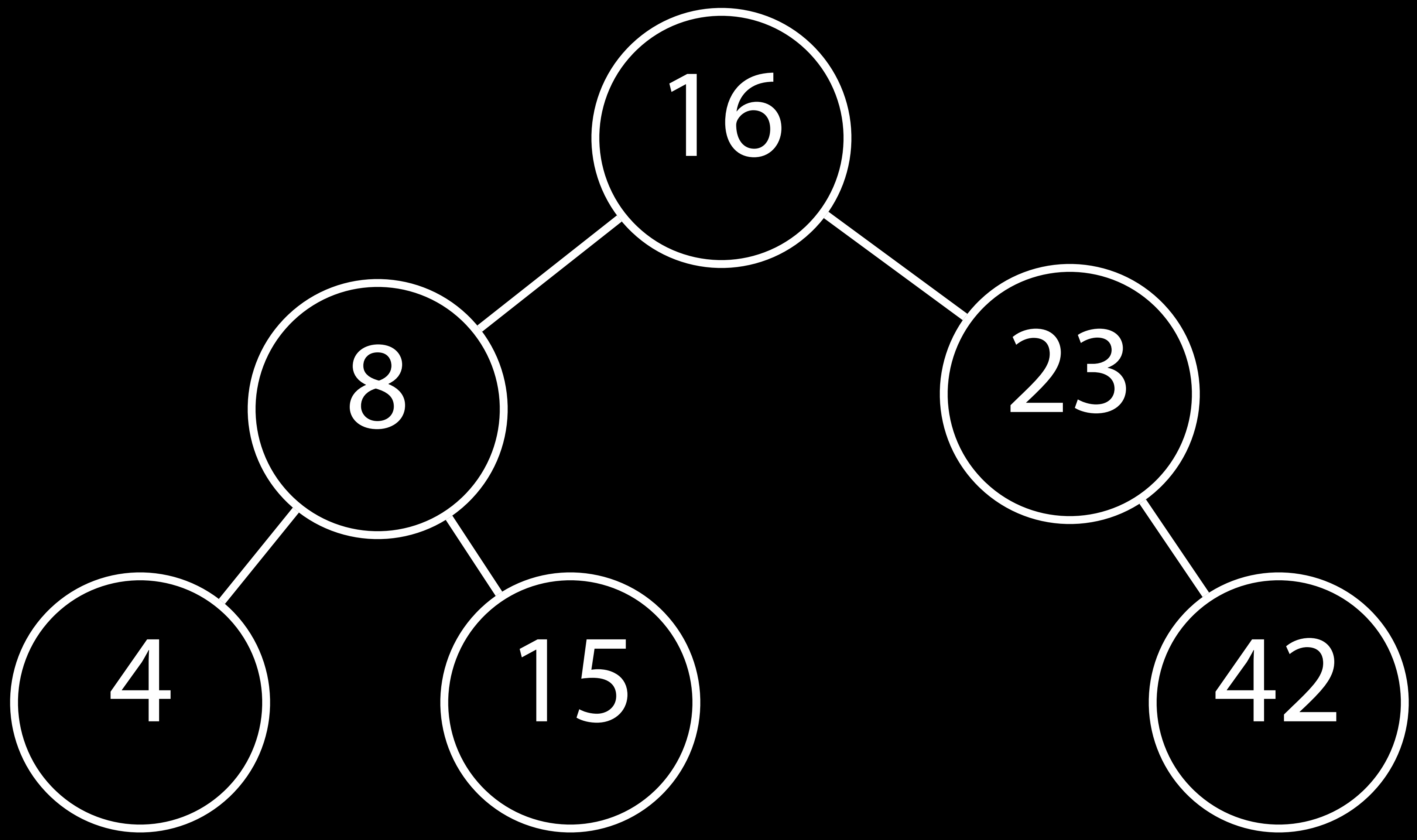


Right Rotation on B



AVL Trees

Binary search tree. For any node, heights of the two child subtrees differ by at most 1.



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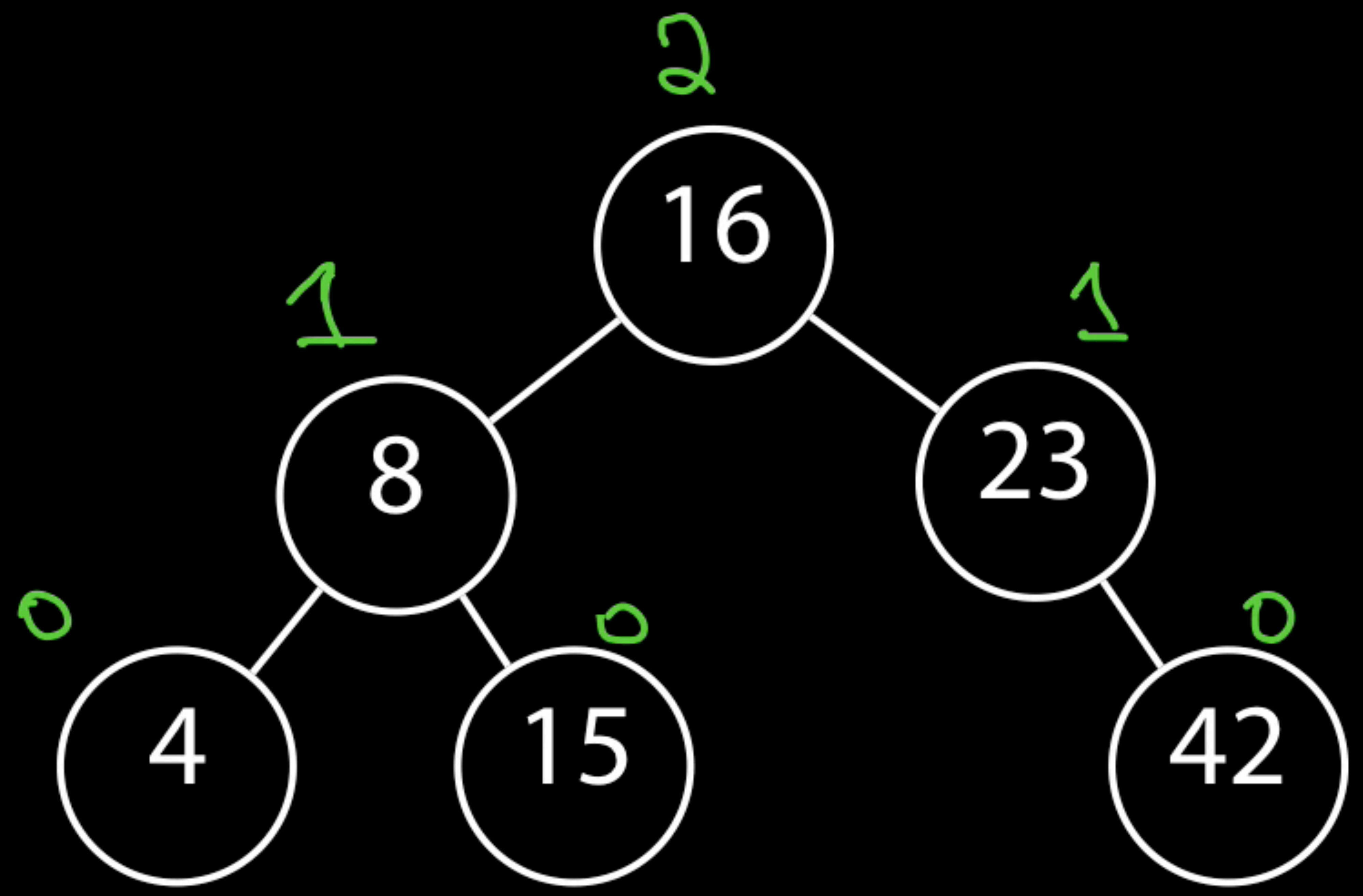
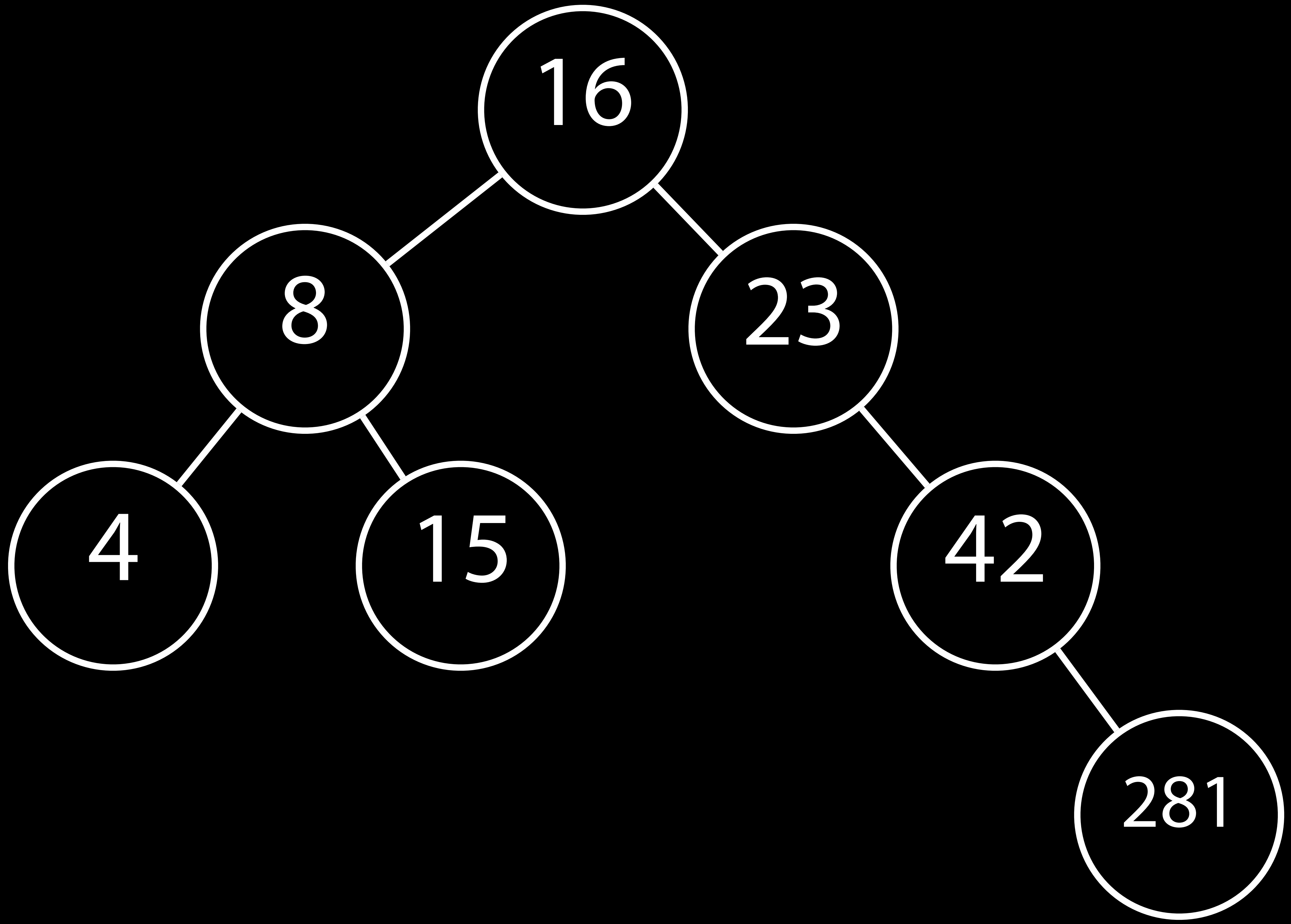
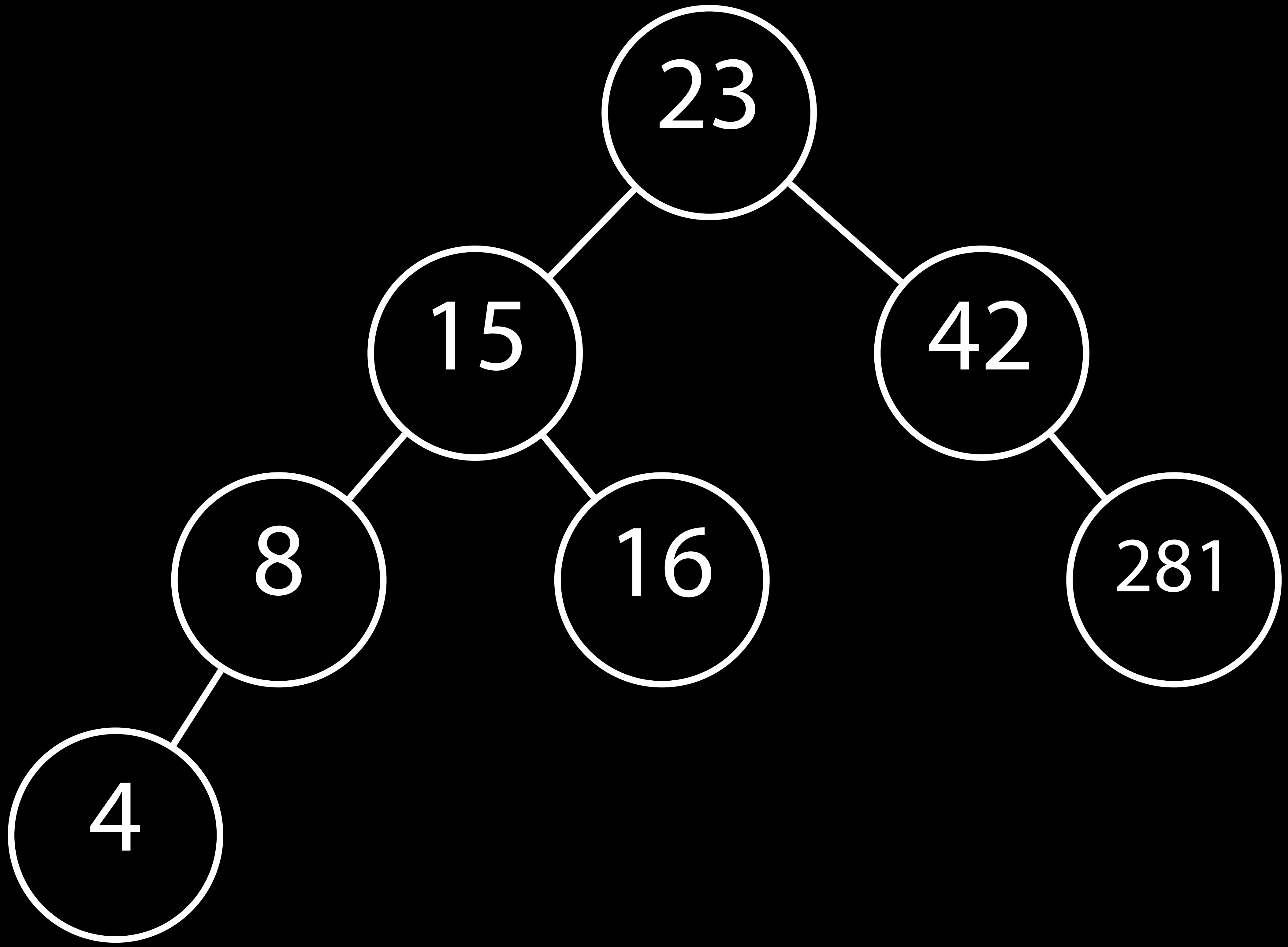
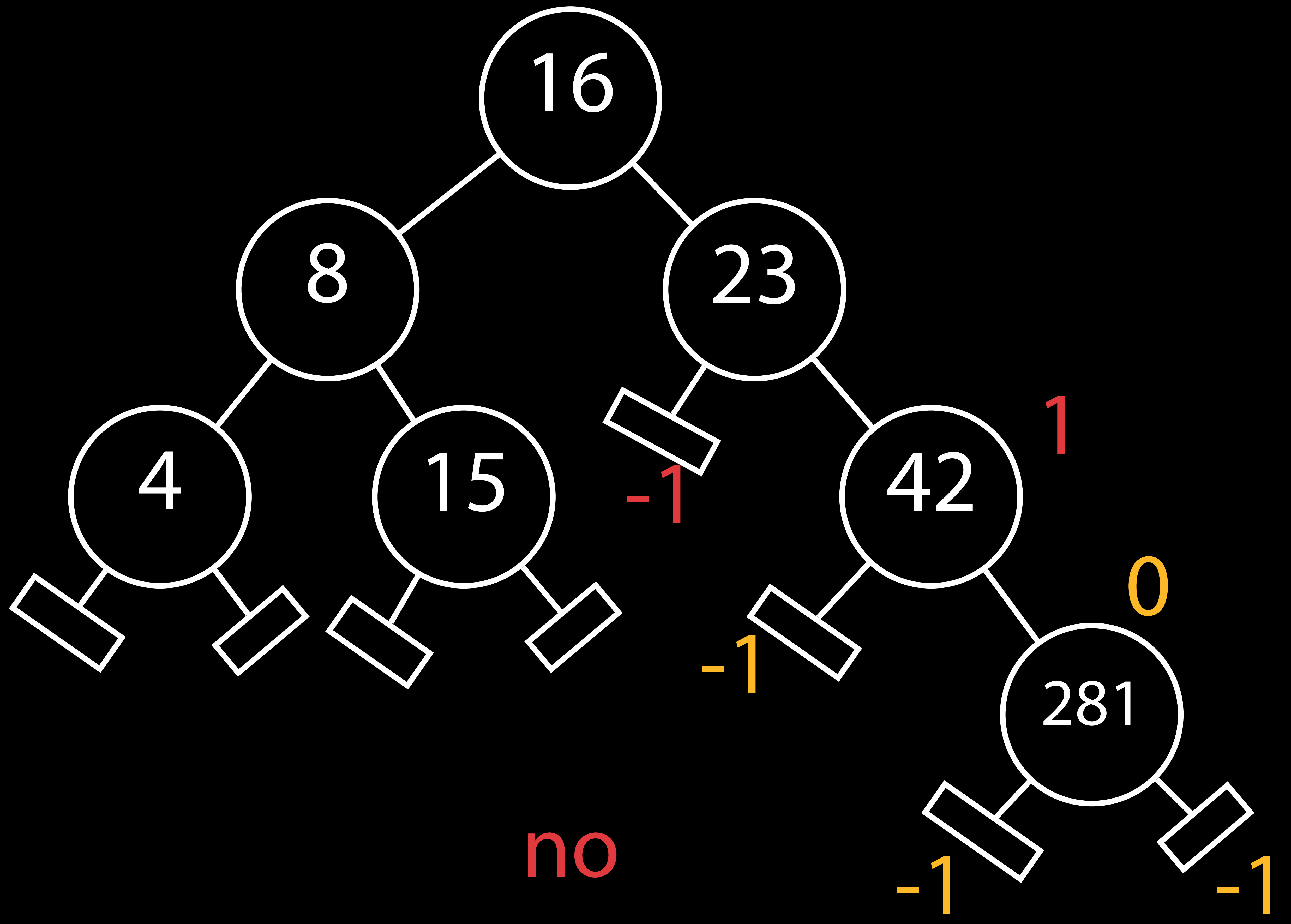


AVL Trees?

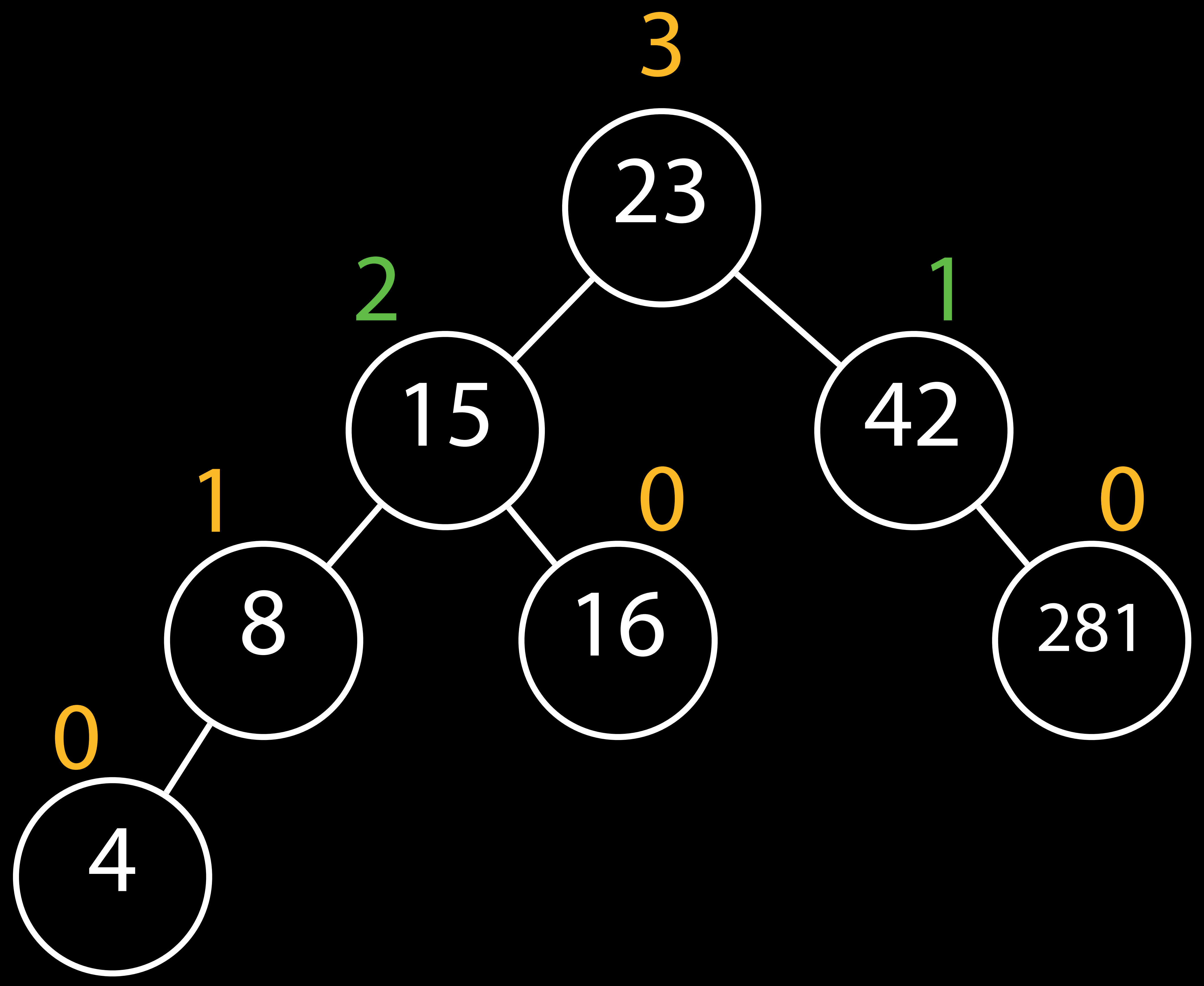




AVL Trees?



no



yes

AVL insert

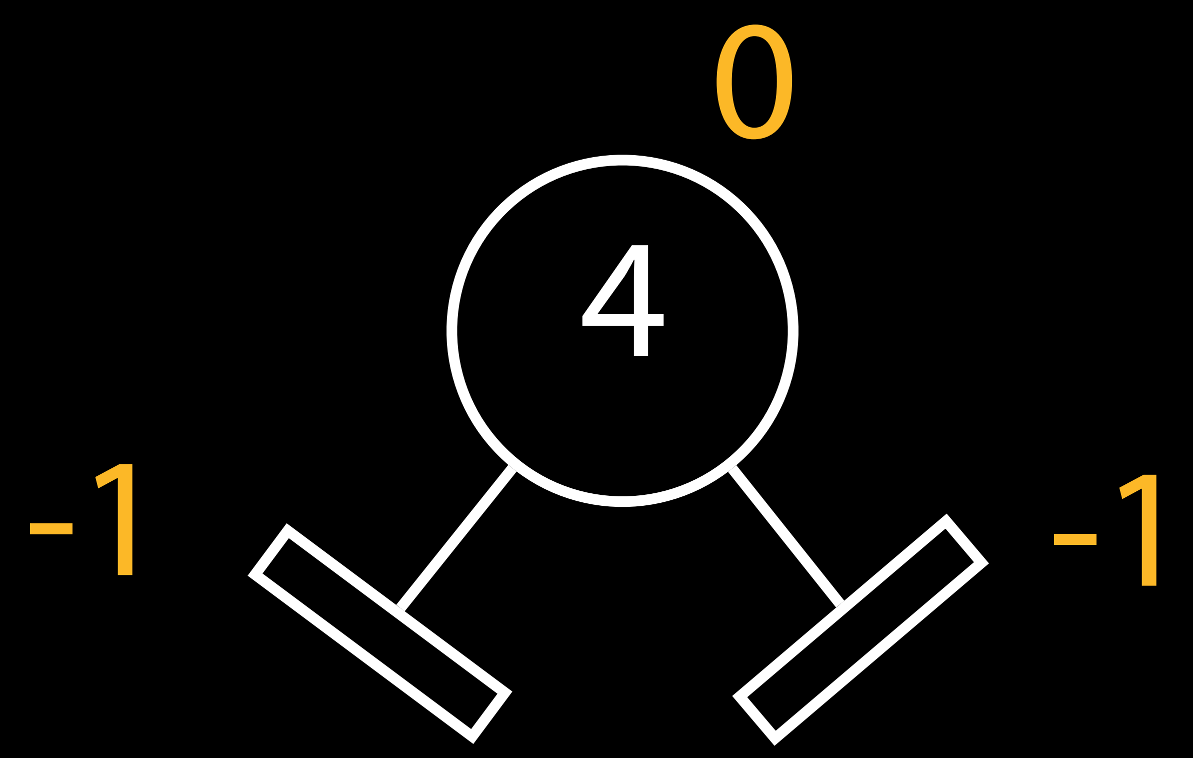
Insert ~~4, 8, 15, 16, 23 and 42~~ 4, 8, 15, 23, 16 and 42.

Insertion

Insert **4**, 8, 15, 23, 16 and 42.

Insertion

Insert 4, 8, 15, 23, 16 and 42.



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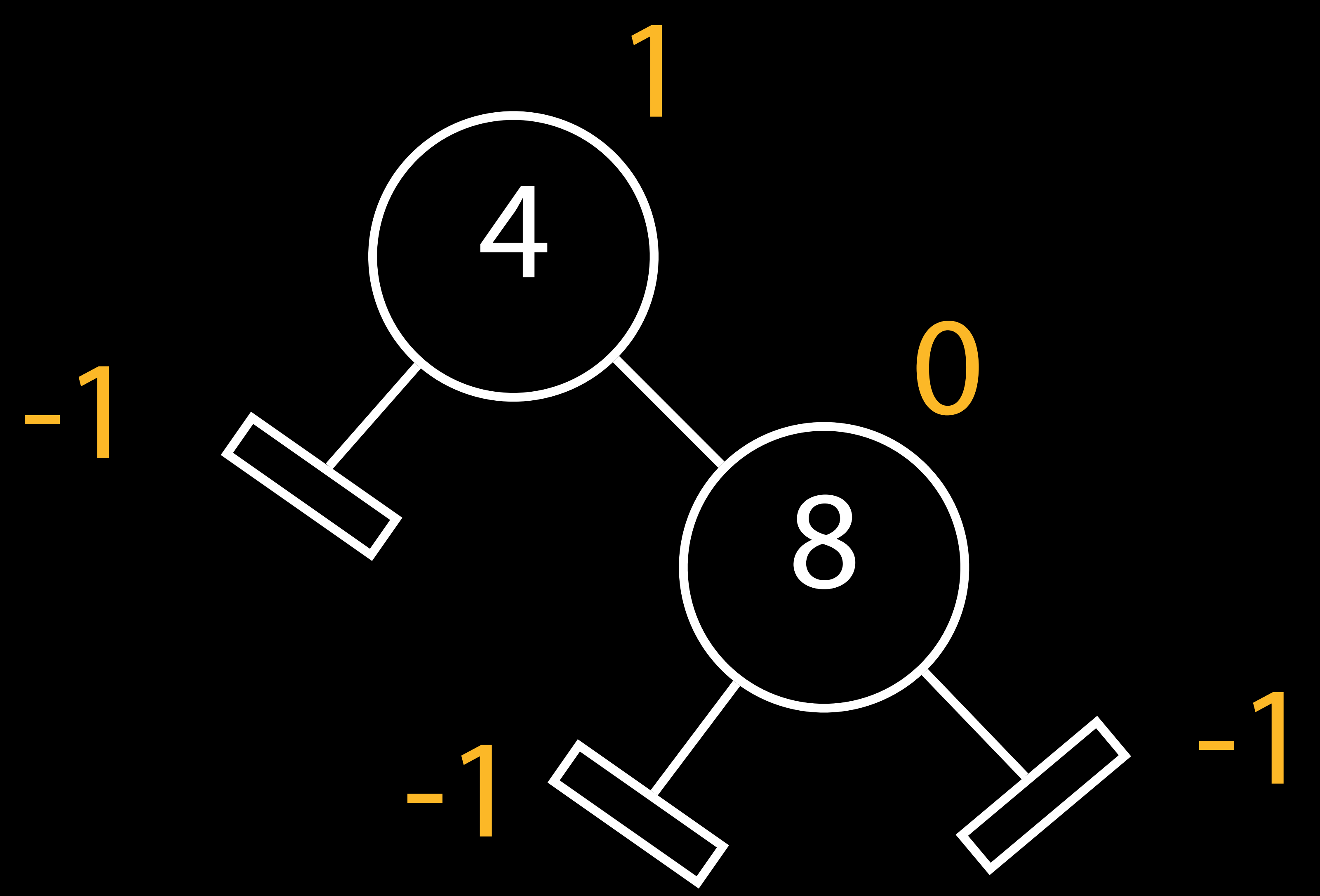

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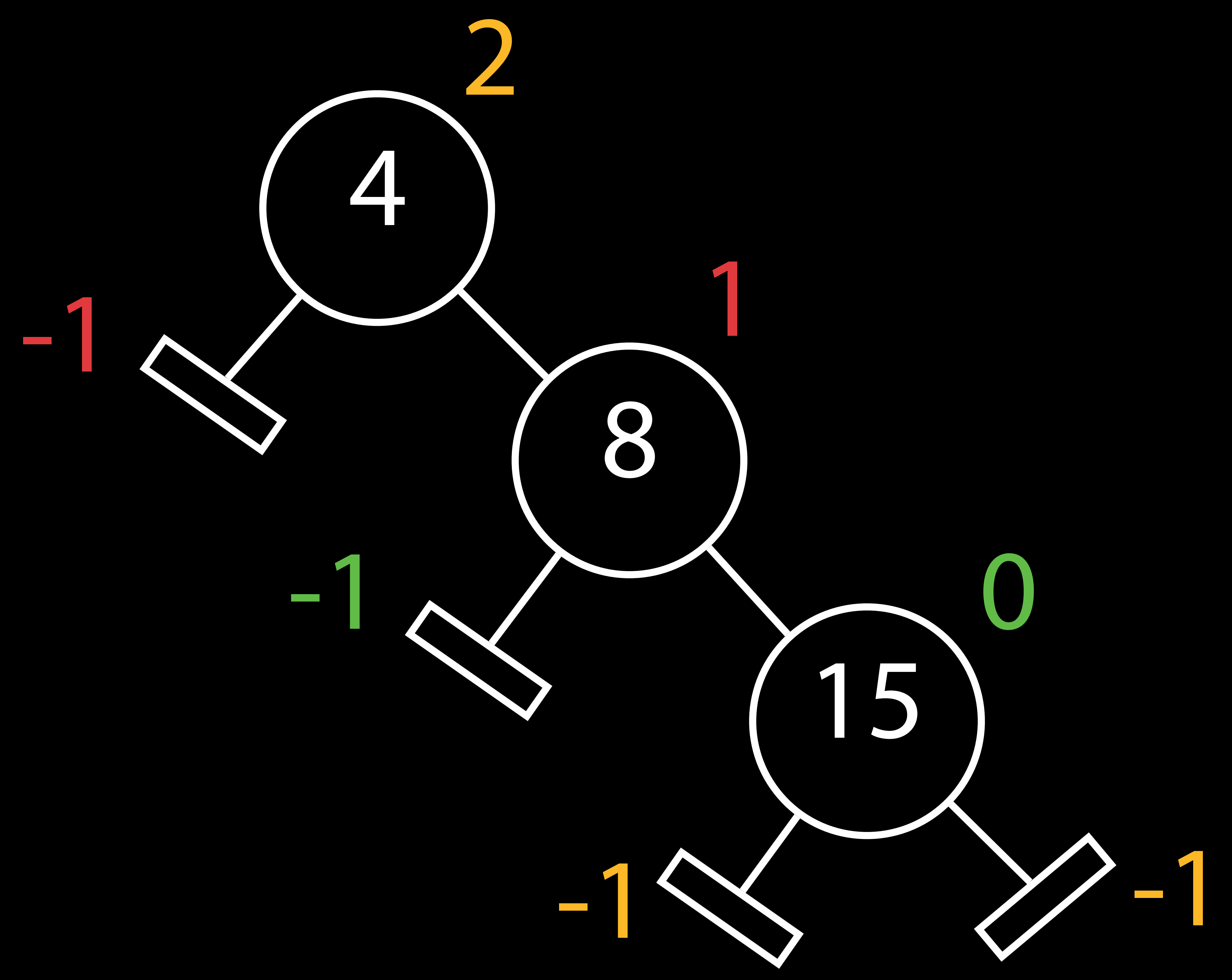

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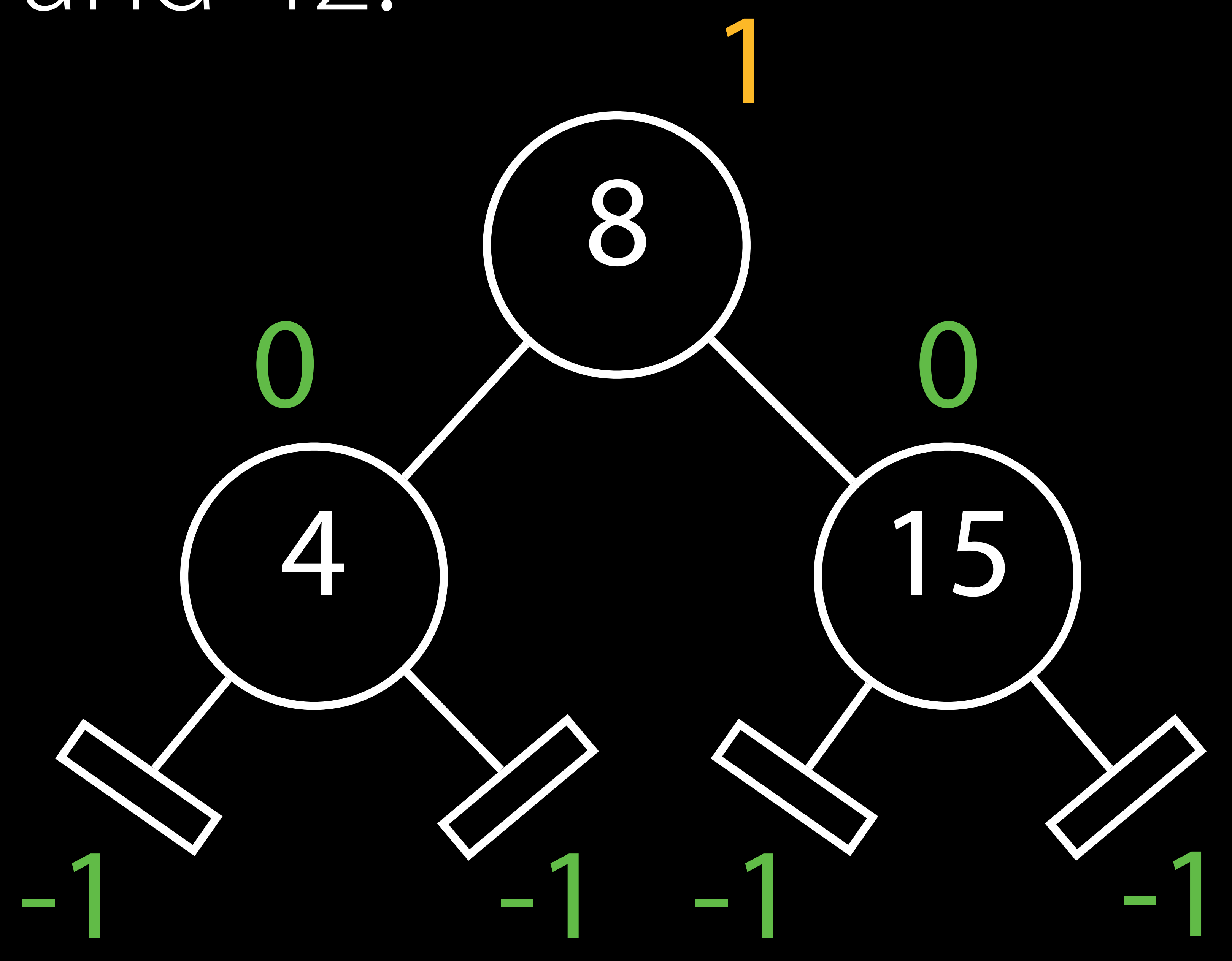

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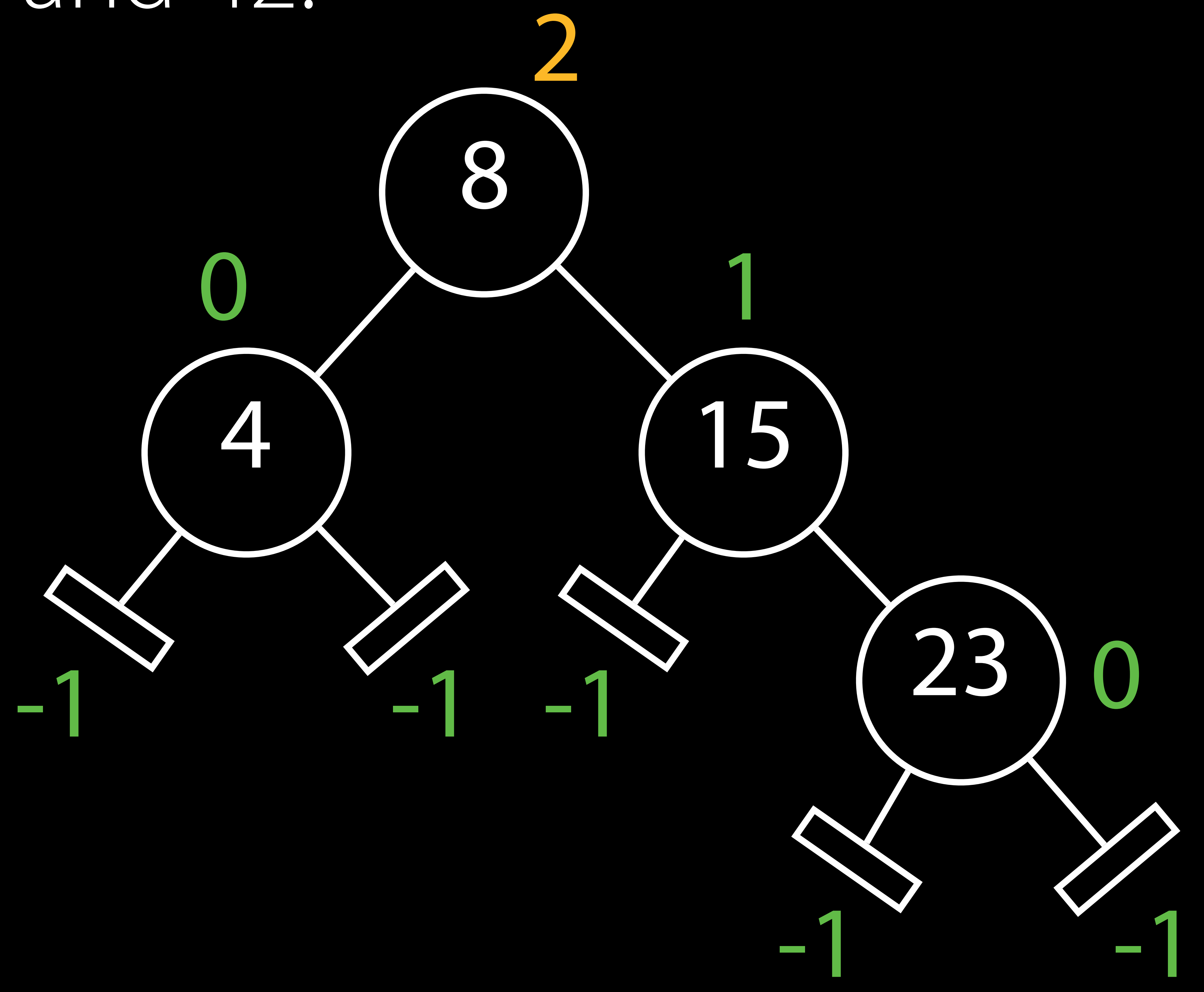

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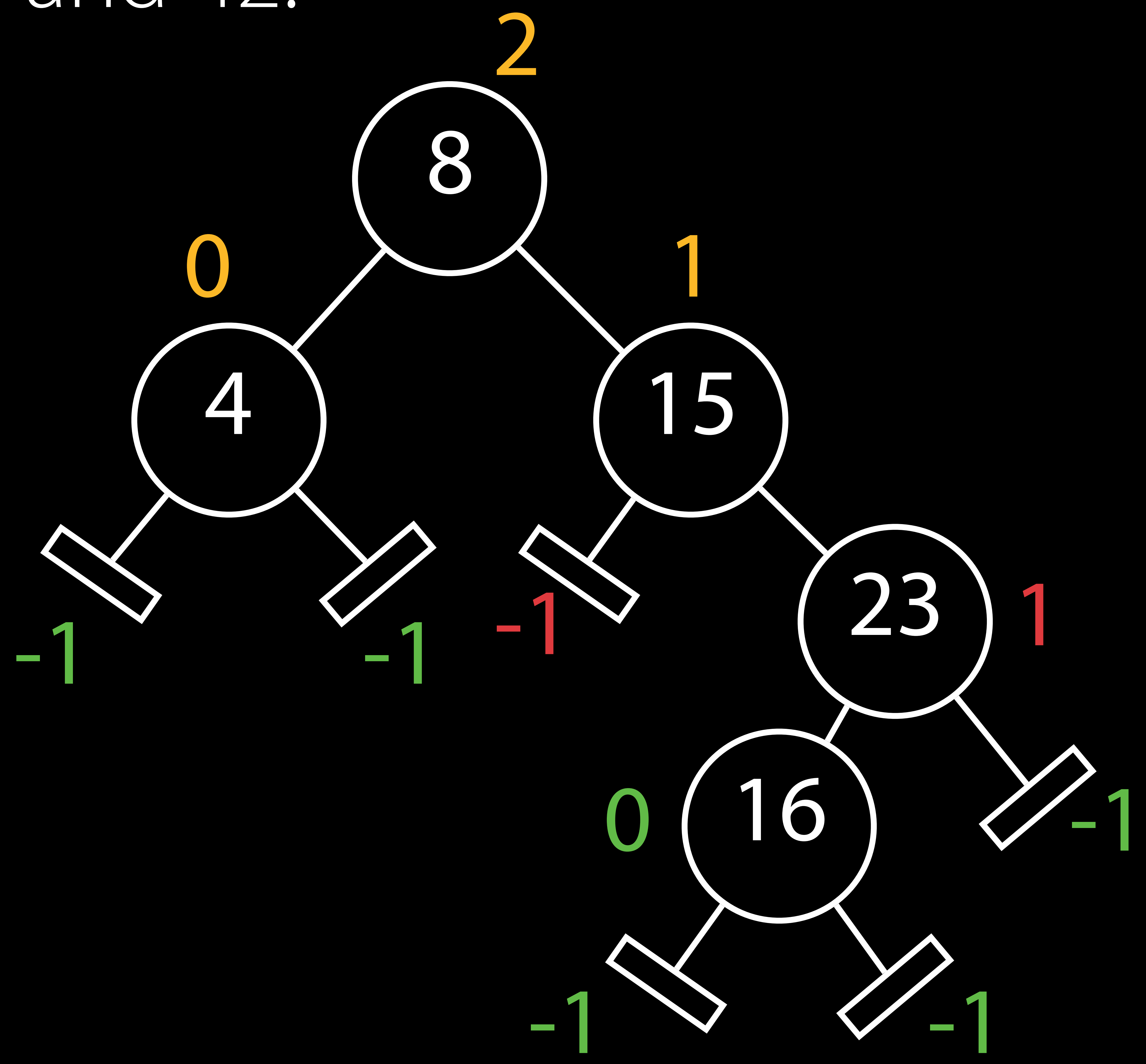

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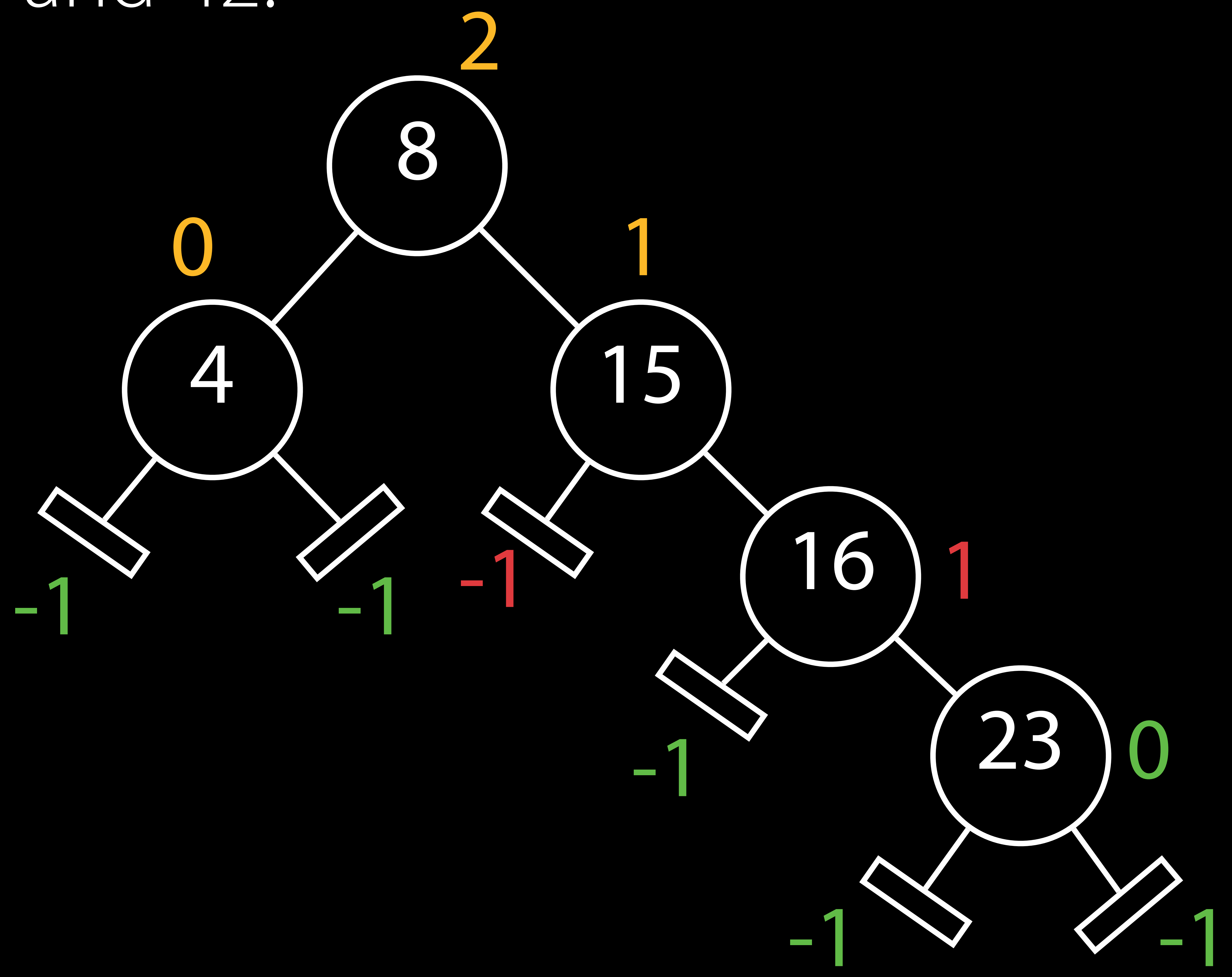

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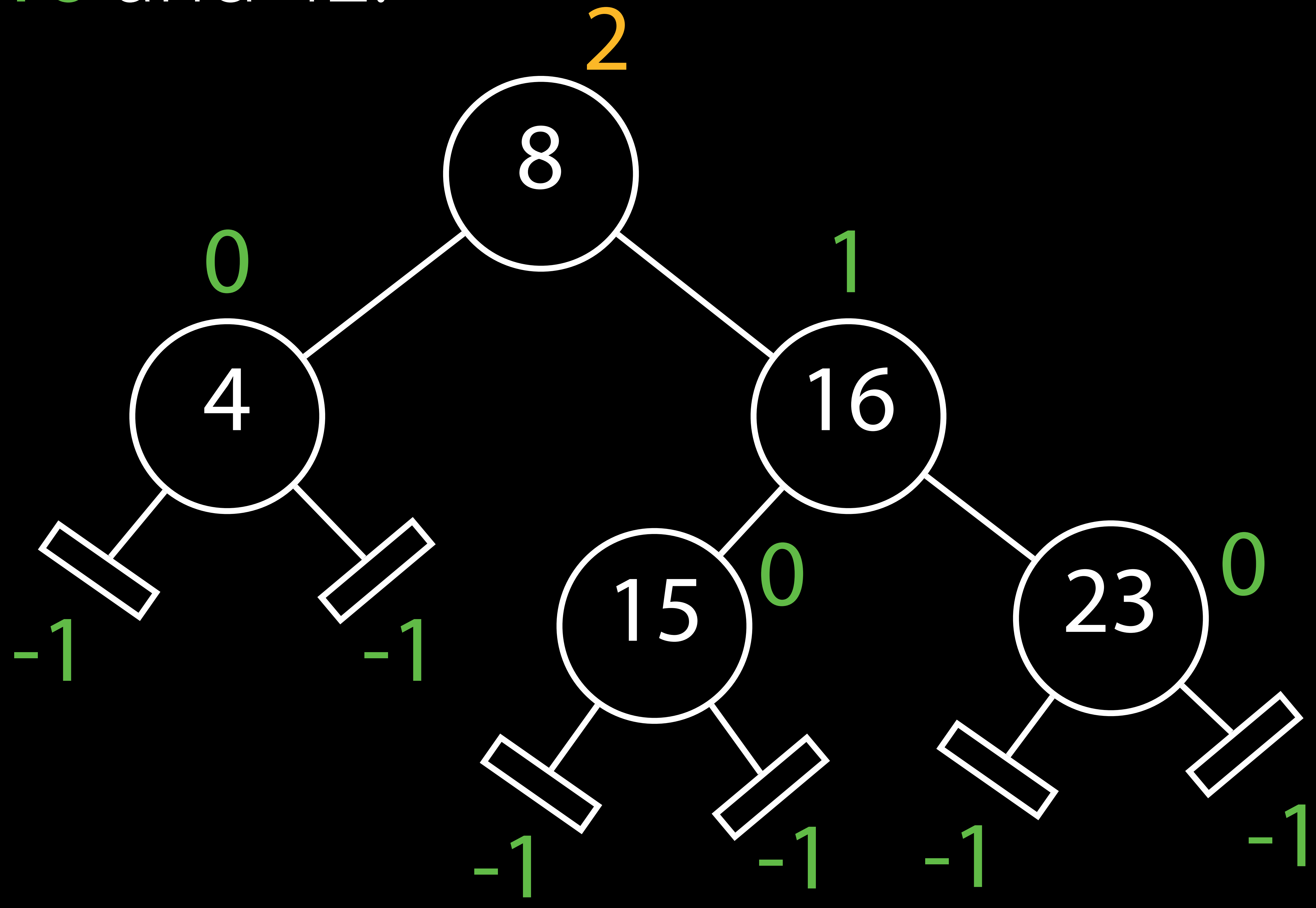

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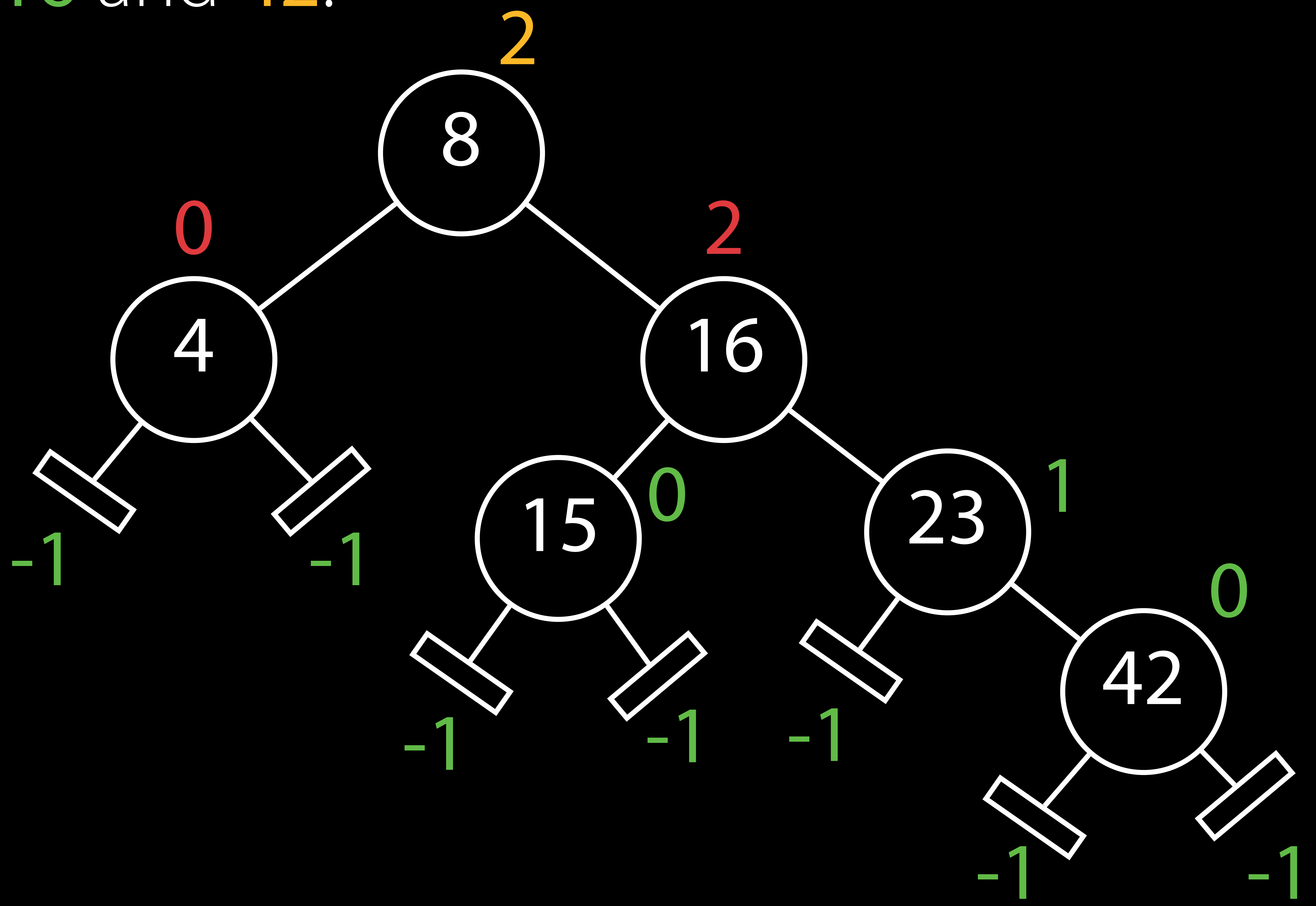

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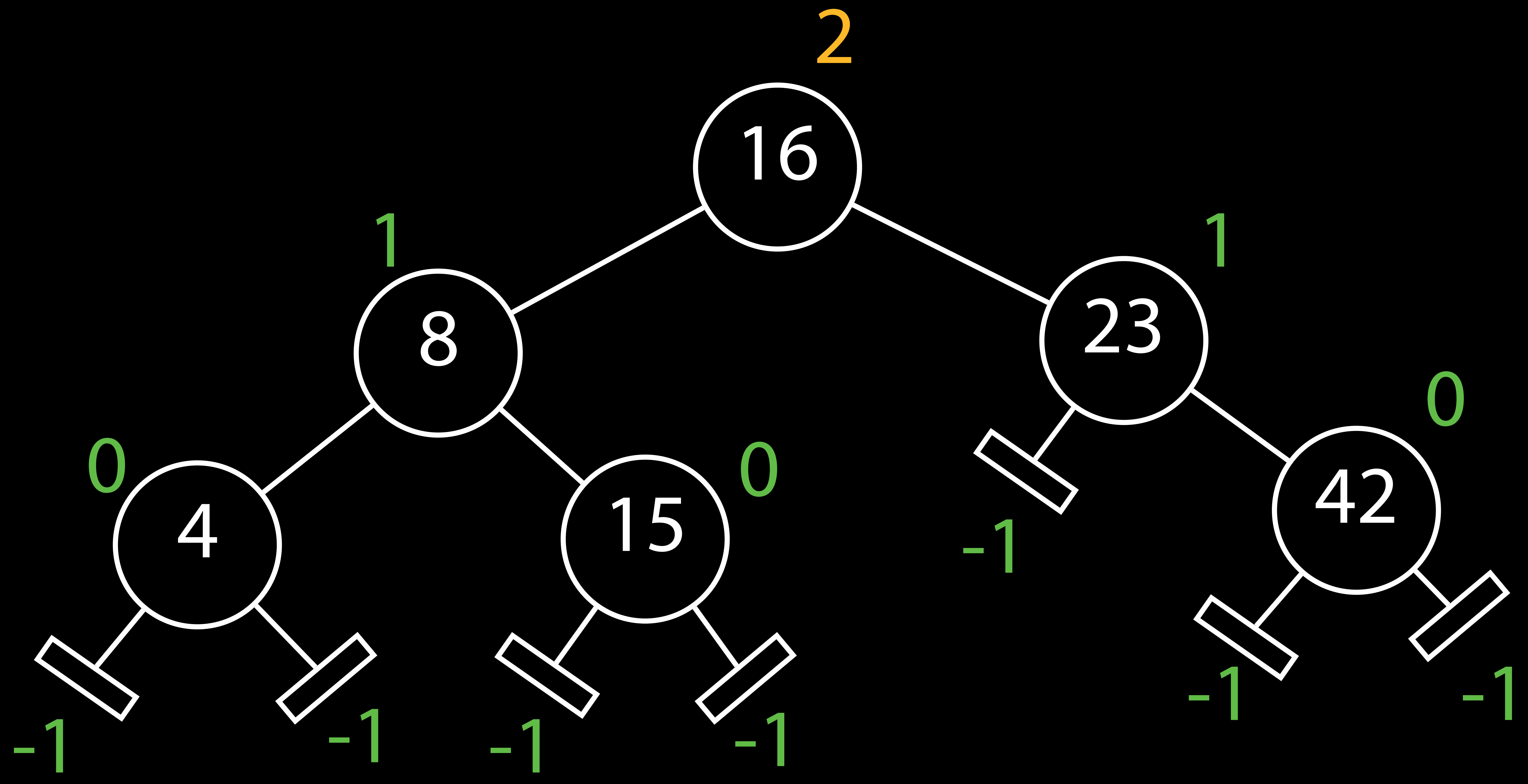


AVL remove

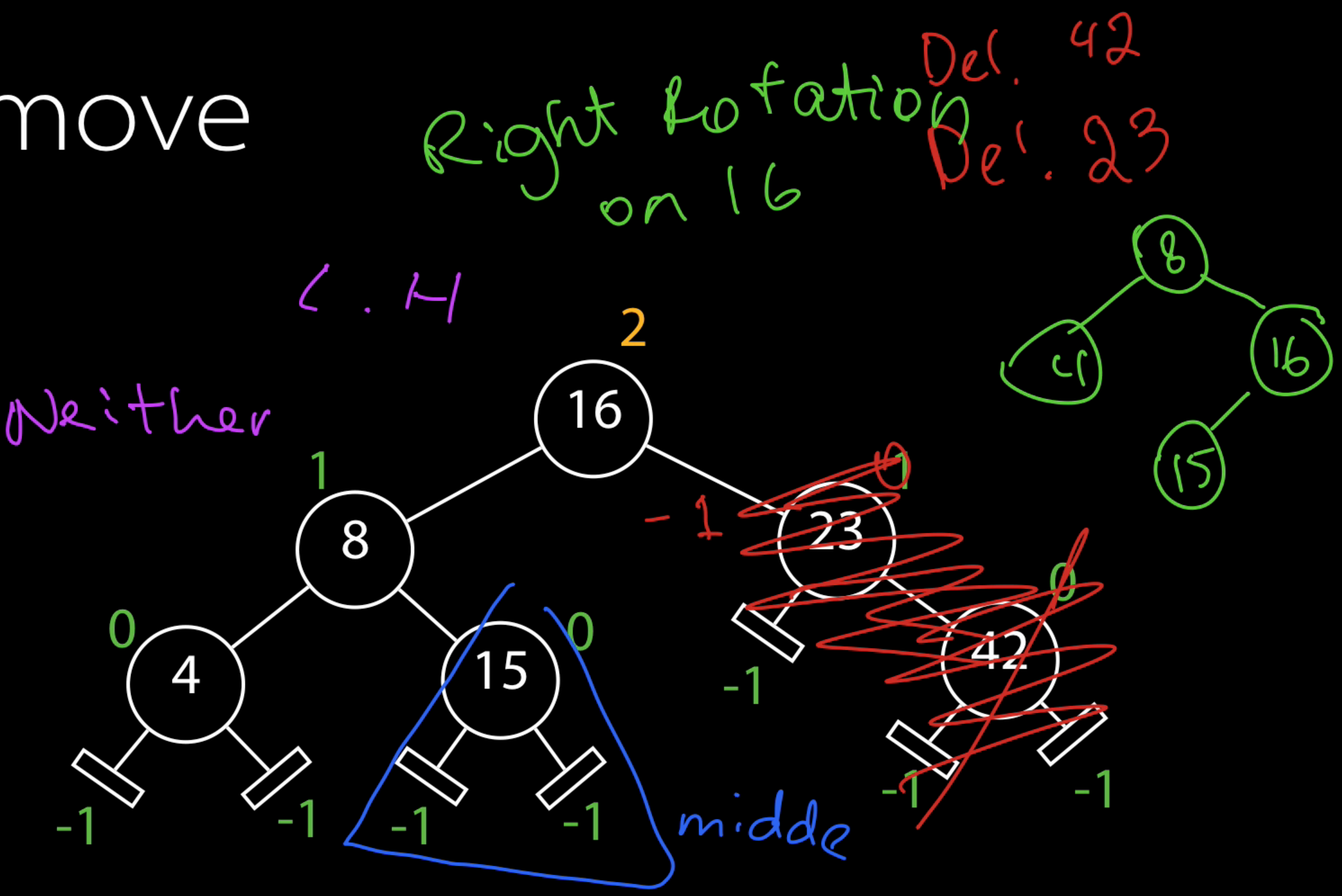
Deleting a leaf node is easy

Deleting a node with one child is easy

To delete a node with two children, replace with in-order predecessor or with in-order successor

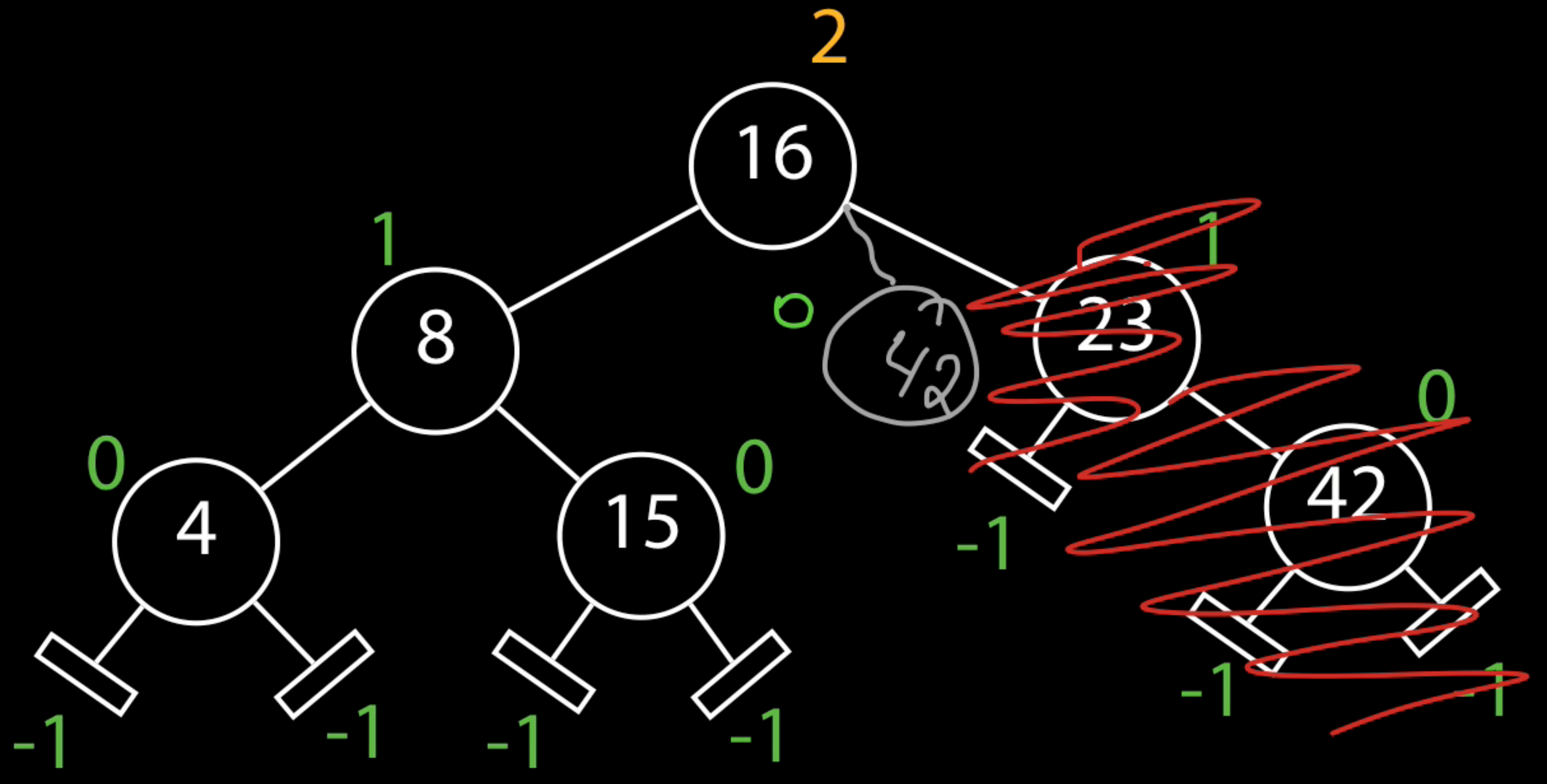
Might need to rebalance after deletion

Remove

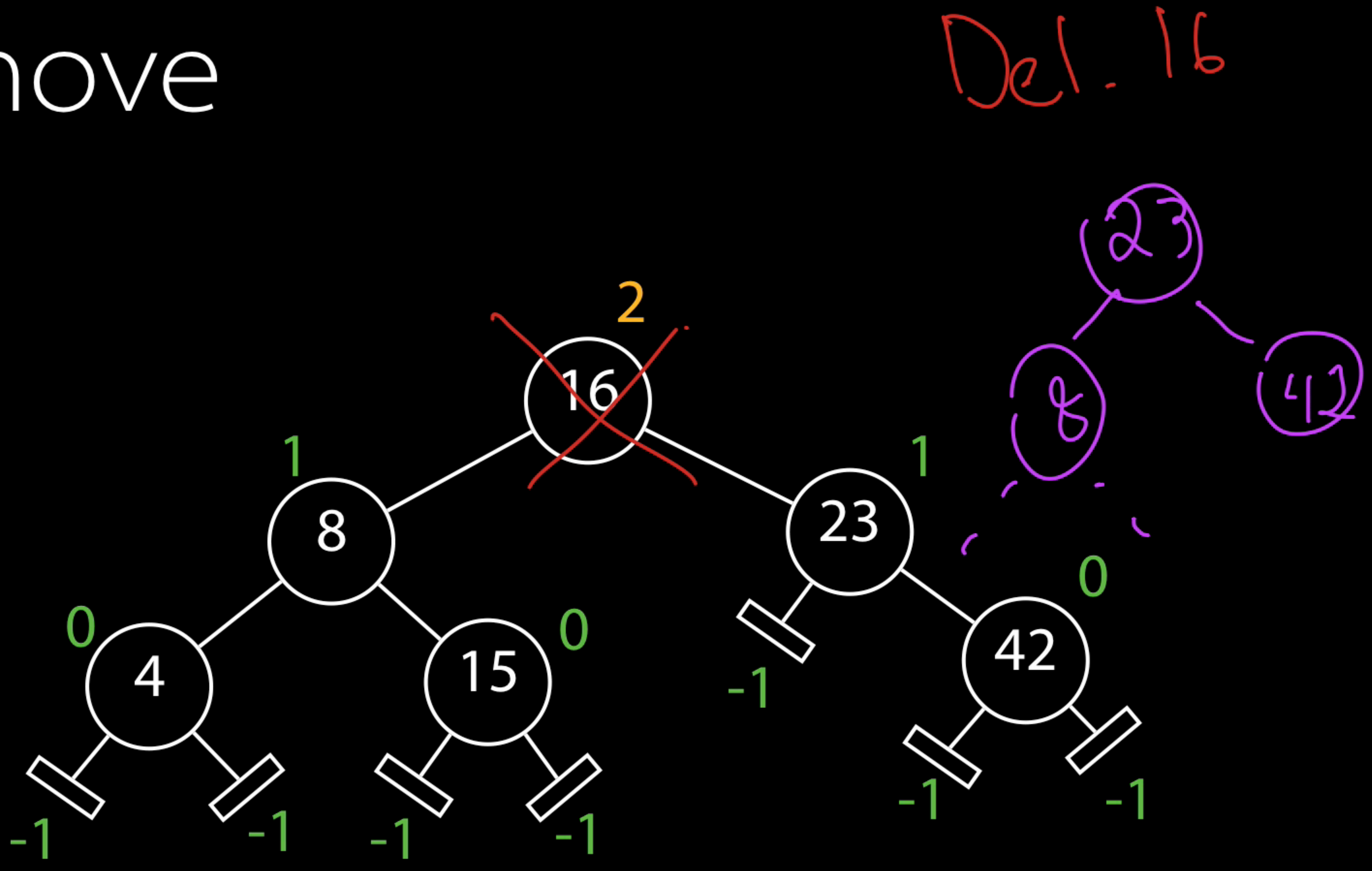

Right Rotation on 16
Del. 42
Del. 23
L. H
Neither
2
16
1
8
0
4
15
23
42
-1
middle

Remove

Del. 23



Remove



AVL tree performance

Searching: $O(\log N)$

Insertion: $O(\log N)$

Deletion: $O(\log N)$

The path from the root to the deepest leaf in an AVL tree is at most $\sim 1.44 \log(N + 2)$

AVL Trees

Given a node in an AVL binary search tree, what is the worst-case complexity of finding the node with the next smaller key?

AVL Trees

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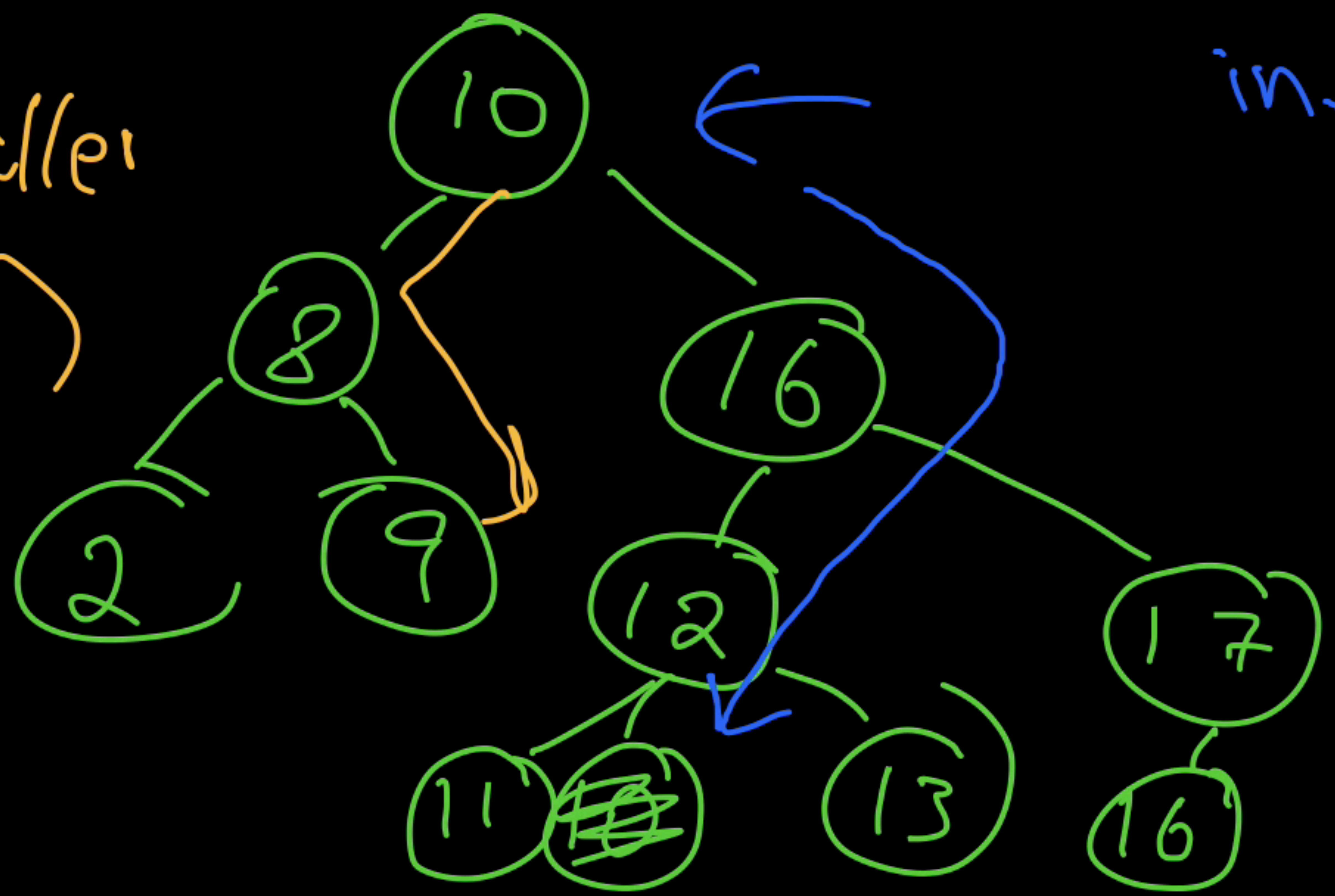
AVL Trees

Given a node in an AVL binary search tree, what is the worst-case complexity of finding the node with the next ~~smaller~~ larger key?

next smaller

O(log n)



in-order successor

O(log N)

AVL Trees

True / false: Deletion of a key from an AVL tree, immediately followed by insertion of the same key always results in the initial AVL tree.

AVL Trees

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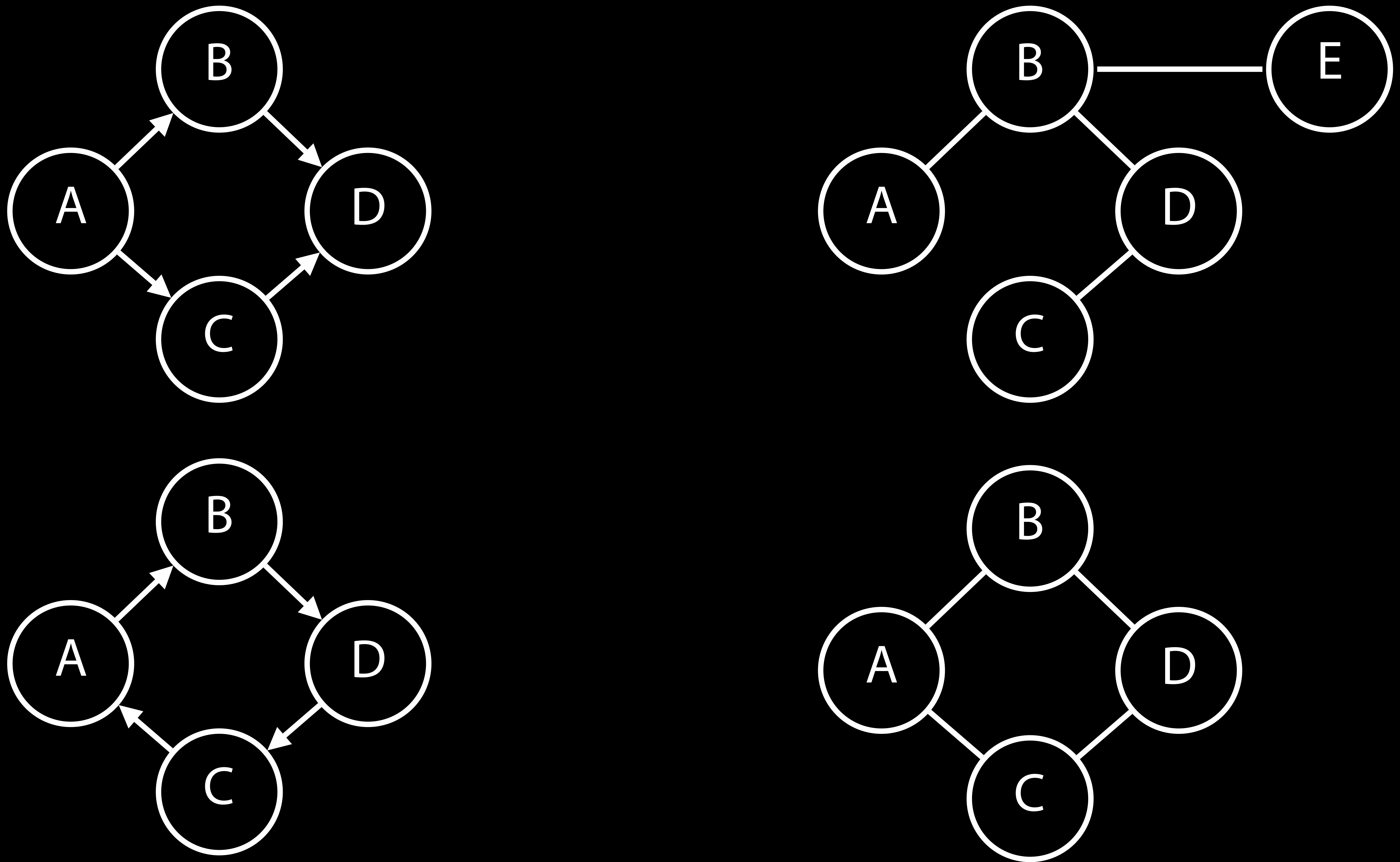
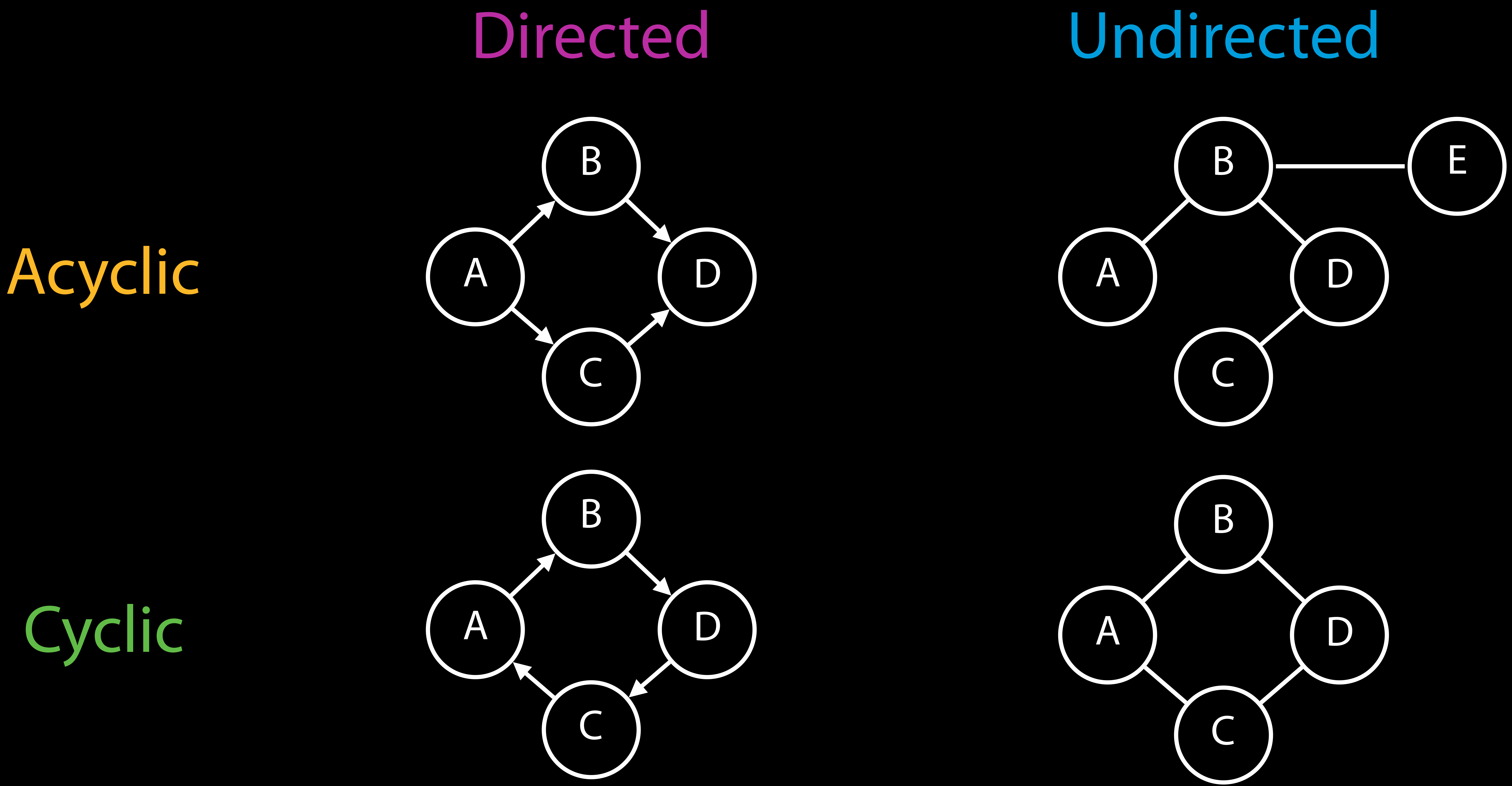
Graphs

Graph Types



Graph Types



Graph terminology

Graph: $G = (V, E)$

Set of **vertices**, a.k.a. **nodes**.

Set of **edges**: Pairs of vertices.

Vertices with an edge between are **adjacent**.

Vertices or edges may have **labels** (or **weights**).

Graph terminology

A path is a sequence of vertices connected by edges.

A cycle is a path whose first and last vertices are the same.

A graph with a cycle is cyclic. A graph without a cycle is acyclic.

Two vertices are connected if there is a path between them. If all vertices are connected, we say the graph is connected.

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A **cycle** is a path whose first and last vertices are the same.

A graph with a cycle is **cyclic**. A graph without a cycle is **acyclic**.

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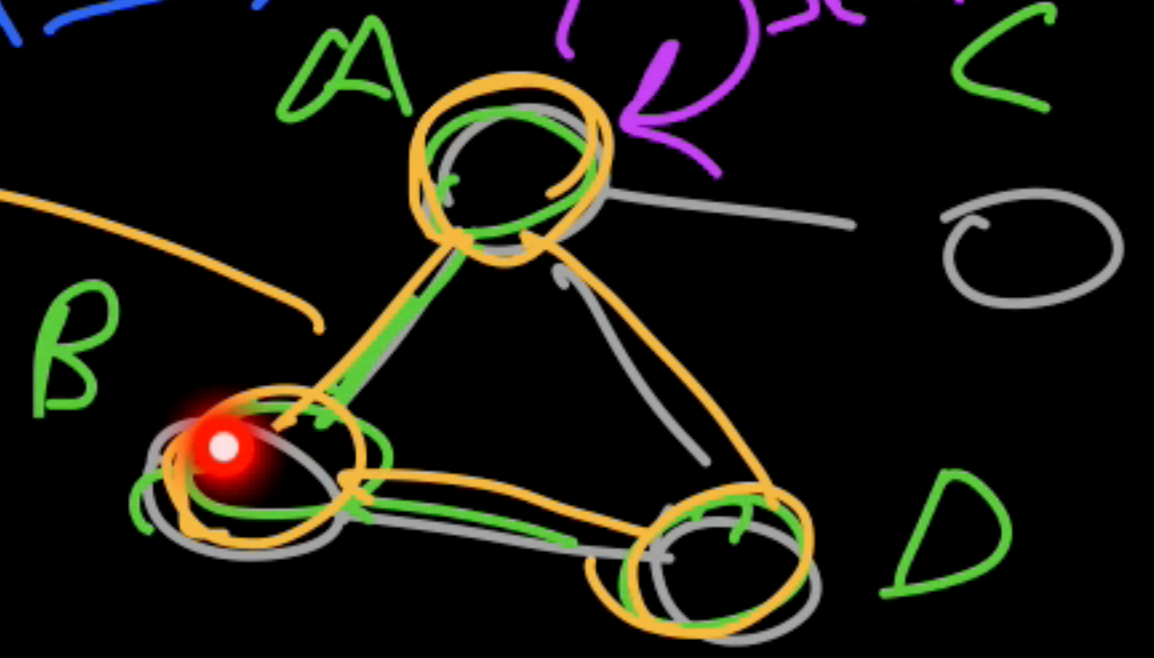
undirected

self-edge

directed →
strongly connected

cycle

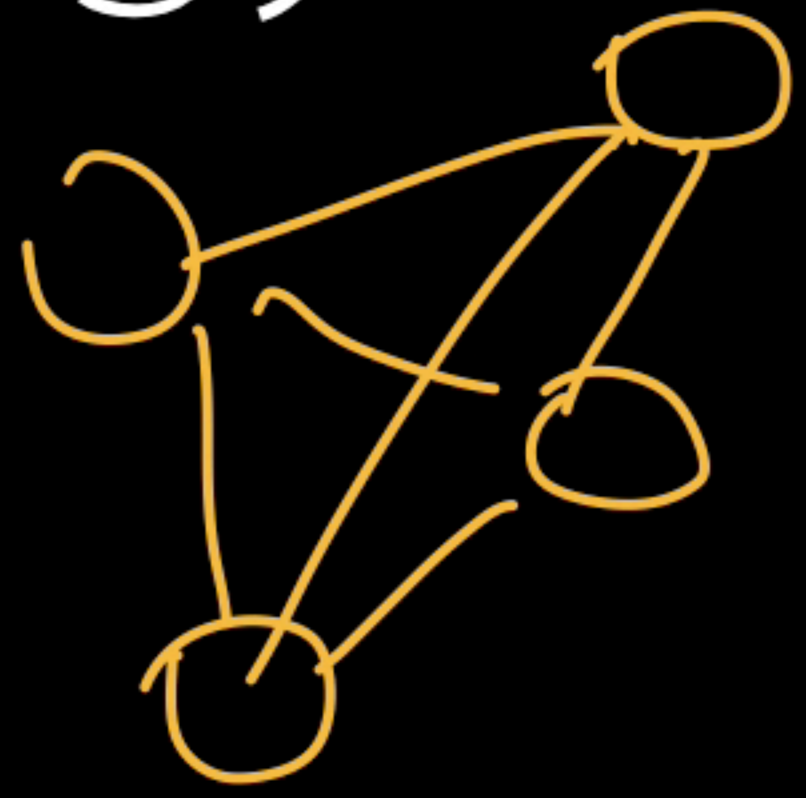


(A, B)
(B, D)

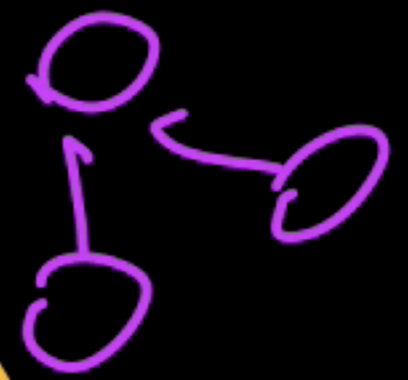
Graph terminology

DAG

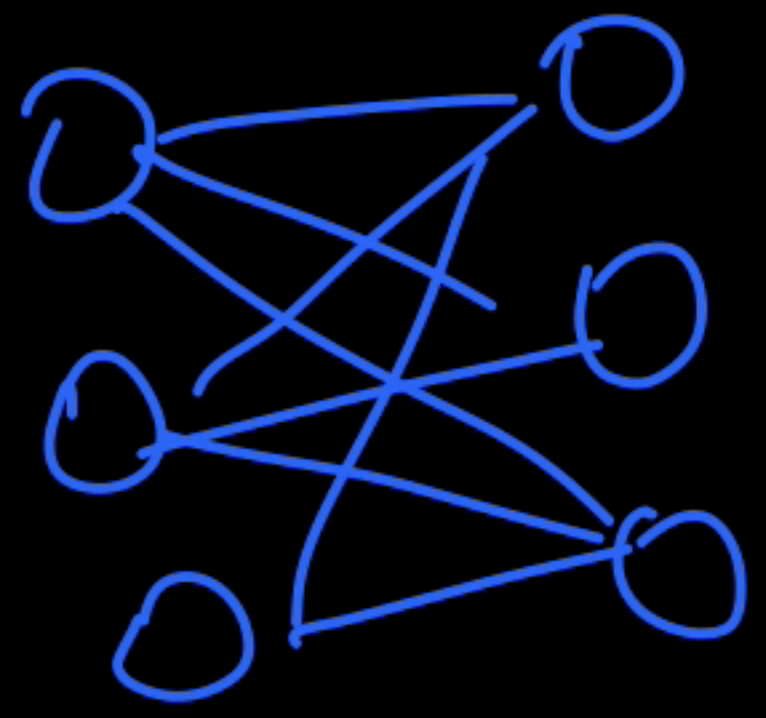
Complete graph



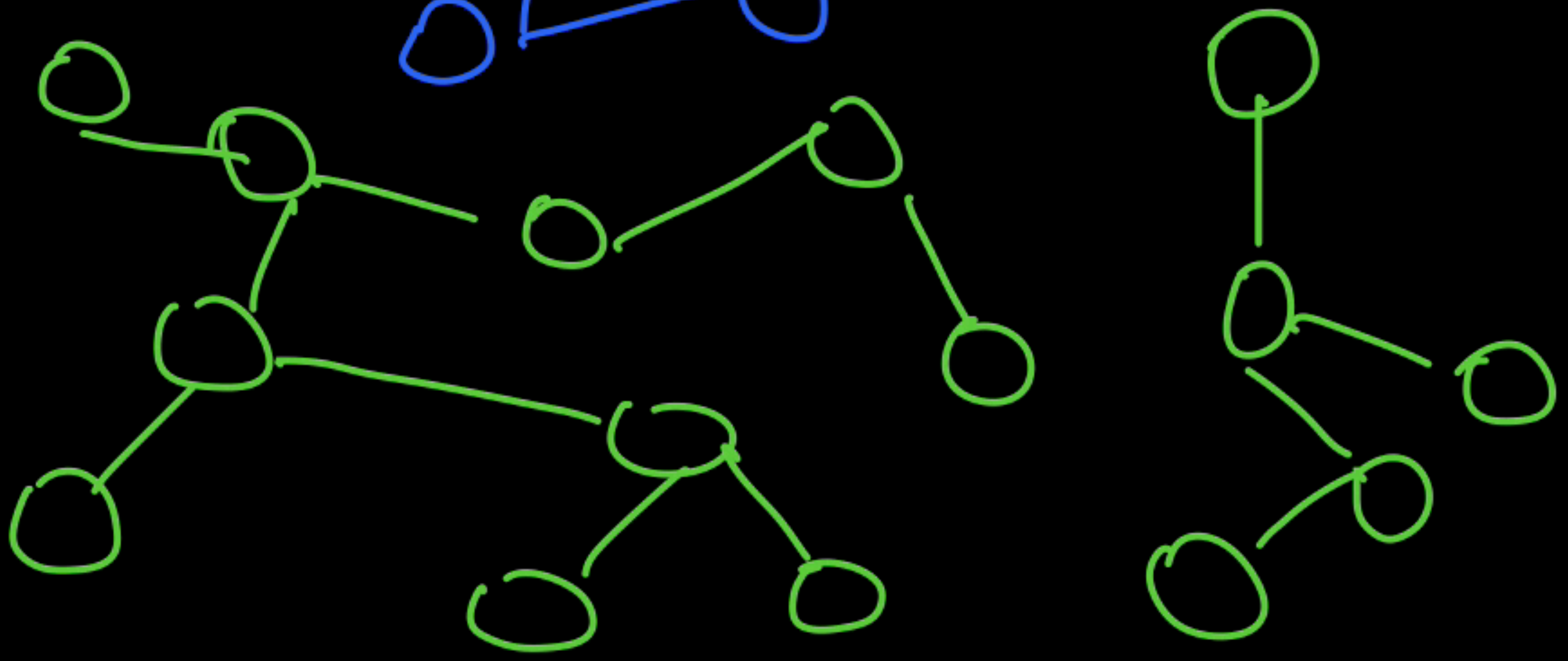
Rooted tree



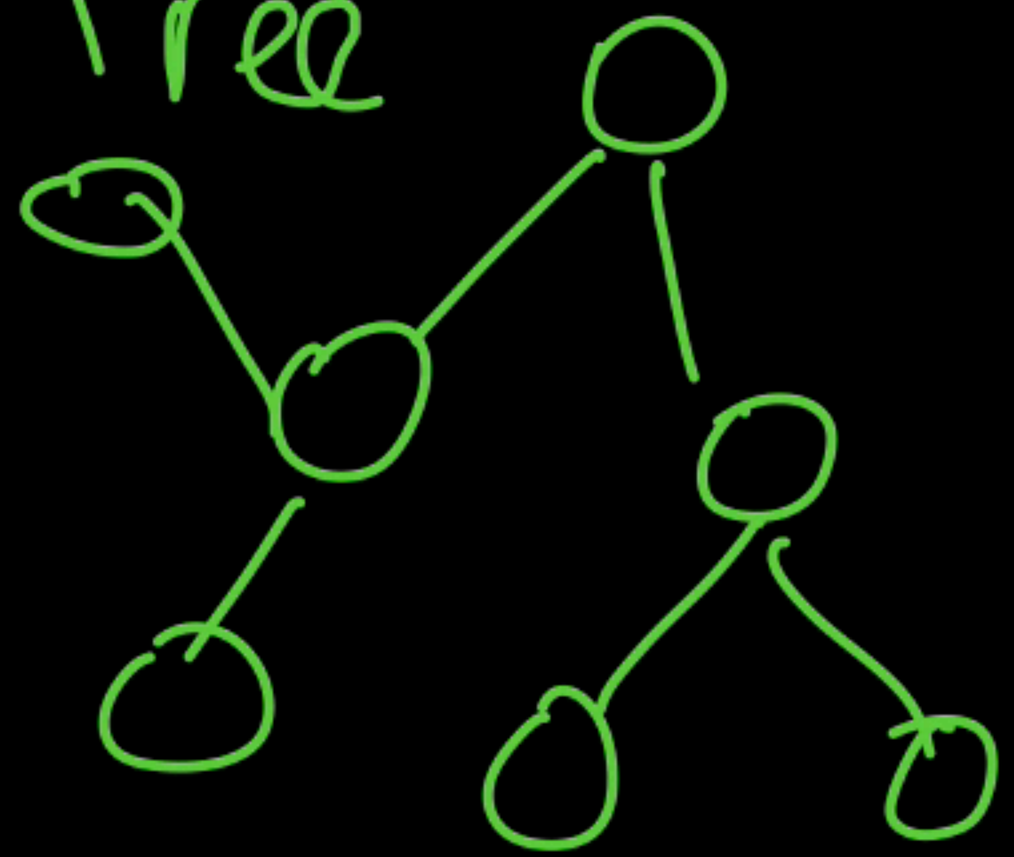
Bipartite



Forest



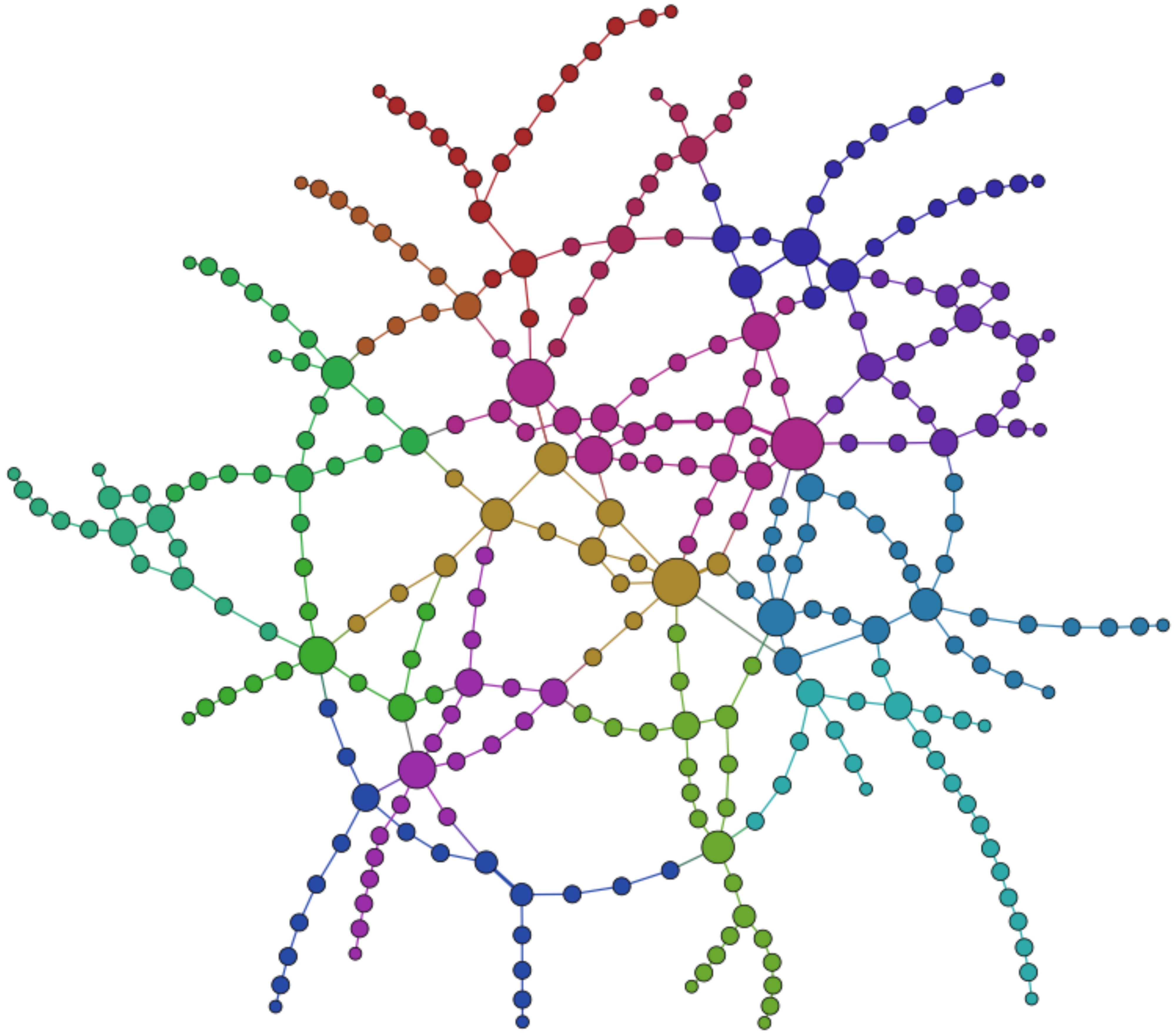
(free) Tree



Graph Example

Introduction to Network Visualization with GEPHI – Martin Grandjean

Examples




facebook
December 2010

Graph representations

$G = (V, E)$

$|V|$ # of vertices

$|E|$ # of edges

sparse: $|E| << |V|^2$

dense: $|E| \approx |V|^2$

Complete graph

$|E| = \Theta(|V|^2)$

Planar graph

(edges cannot cross)

$|V| \geq \Theta(|E|)$

Graph representations

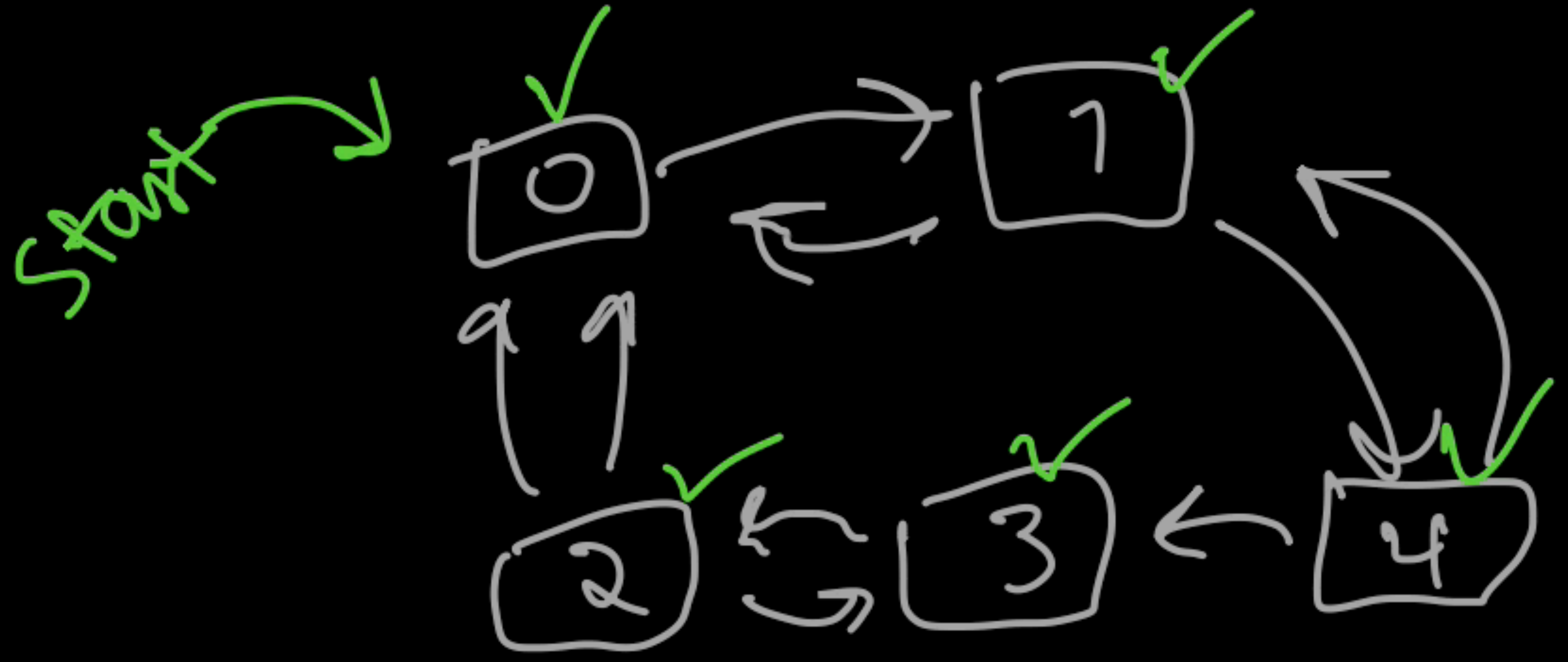
Graph traversal

Pre-order depth-first traversal

Post-order depth-first traversal

Breadth-first traversal

Graph traversal



Stack

1. Pre-order depth-first traversal

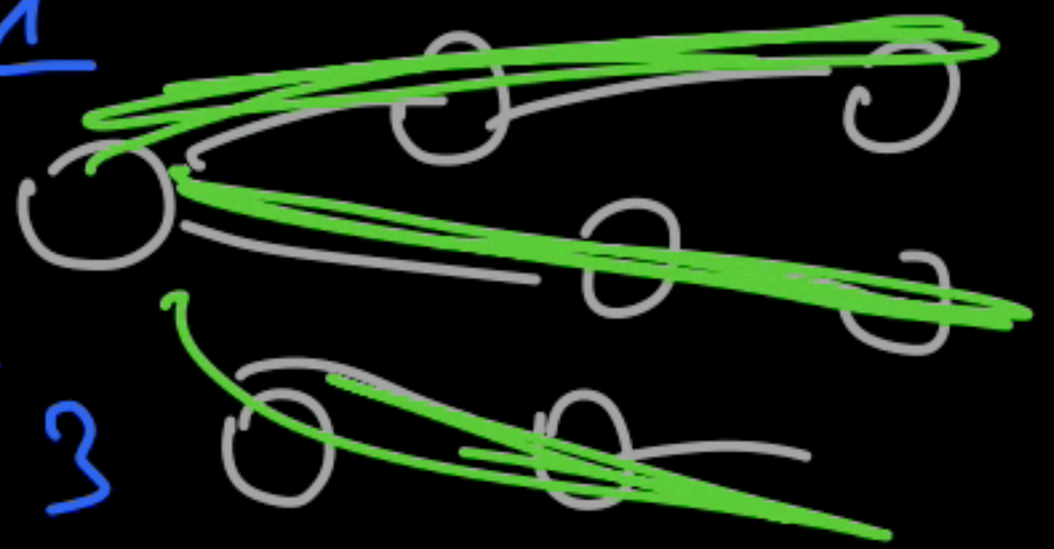
Post-order depth-first traversal

Breadth-first traversal Queue

Level-order

0 1 2 4 3

visit everything at level 1
level 2
level 3



visit 0
visit 1
visit 4
visit 3
visit 2

dfs(0)
dfs(1)
dfs(2)
dfs(4)
dfs(3)
dfs(2)

Silicon Valley

http://www.hbo.com/silicon-valley

Q&A